Abstract

Credit risk may be correlated because the observed and unobserved drivers of credit risk happen to be correlated, or because they are related through contagion. We identify contagion by assuming that contagion takes time. Bank credit risk is measured by the proportion of problem loans in sectors of Israel’s banking system. Dynamic factor models are estimated for seven main sectors in which contagion is hypothesized to take place between sectors. The risk factors identified include sector-specific as well as systemic variables. Credit risk is correlated both statically and dynamically between sectors due to contagion, common risk factors, and correlated risk factors. The estimated model is used to simulate these phenomena.

Keywords: bank credit risk, contagion, correlated risk
Introduction

First generation credit risk models\(^1\) ignored the fact that credit risk might be correlated. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) used asymmetric information theory to argue that because agency costs, moral hazard and adverse selection are anticyclical, credit risk should be anticyclical. This theory predicts that credit risks should be correlated through the business cycle. Empirical support for this prediction has come from Blume and Keim (1991) and Jonsson and Fridson (1996) who showed that bond defaults are anticyclical. Lennox (1999) used data on company failures and extended the “Altman Model” to take account of the business cycle. Following Nickell et al (2000) there have been a number of attempts to explain credit risk ratings by cyclical and other macroeconomic factors\(^2\). Finally, Carling et al (2007) use proprietary data on bank credit to show that corporate failure depends on the business cycle and other macroeconomic phenomena.

More recently contagion has been proposed as an additional channel inducing correlated risk. The copula model (Li 2000) has been widely used to model correlation in credit risk. However, this mechanical model has no economic structure and does not distinguish between correlation induced by macroeconomic or systemic factors on the one hand, and contagion on the other. More recently (Giesecke and Weber 2004, Egloff et al 2007, Horst 2007) attempts have been made to introduce economic structure by distinguishing between correlation induced by common risk factors, which tend to be macroeconomic, and correlation induced by contagion. The latter is induced by counterparty risk\(^3\). If debtor A owes money to creditor B, and A defaults, this might induce B to default, which might induce C to default if B owes money to C etc. This class of models not only requires micro data, it also requires data on counterparty risk relationships. Such data tend to be proprietary, which naturally inhibits research. Indeed, whereas the business cycle mechanism has been studied extensively, there are no empirical studies\(^4\) on the counterparty risk model of contagion.

\(^1\) Such as Merton’s (1974) Structural Model, its first passage extension by Black and Scholes (1976), and Jarrow and Turnbull’s (1995) Reduced Form Model.
\(^3\) These models are not without conceptual difficulty. For example, the counterparty risk lattice is usually assumed to be exogenous and counterparties are selected randomly. The choice of counterparties is likely to depend on credit risk. Indeed, the rush to mitigate counterparty risk exposure is likely to trigger a credit risk cascade.
\(^4\) There may, of course, be unpublished proprietary studies.
Essential micro data are unavailable, either because debtors are private companies, or because default histories are not public information. In such cases market-based credit risk models are not feasible. In the case of bank credit risk confidentiality prevents the publication of information on individual customers. These proprietary data are no doubt analyzed by the banks themselves and individual customers are profiled in terms of their credit risk. However, the results naturally remain unpublished. Information on individual credits becomes public if the credit is traded in the market for swaps. However, this is typically the tip of the iceberg. As noted by Dermine and de Carvalho (2005), data scarcity explains why empirical studies of bank credit risk are rare. This applies a fortiori in the study of credit risk models in which contagion occurs through counterparty risk.

Although micro data are not published on bank credit risk, most central banks publish aggregated data on problem loans for individual banks or for the banking system as a whole. These data are sufficiently aggregated to prevent the disclosure of information on individual debtors. For example, the Bank of Israel publishes data on problem loans by economic sector for the banking system as a whole but not for individual banks, and it publishes data on problem loans as a whole for individual banks. To our best knowledge such data have not been used before to shed light on bank credit risk and its determinants. We therefore use these data to estimate factor models for bank credit risk in Israel in which contagion occurs between economic sectors rather than between counterparties. We propose an empirical factor-based credit risk model that serves as an alternative to market-based models. Also, whereas market-based models are concerned with the credit risk of individual firms, our proposed factor-based model is concerned with systemic credit risk.

We argue that there is more to contagion than counterparty risk. Contagion may also be induced by inter-industry or inter-firm trade. Adverse shocks might transmit themselves between different sectors of the economy through vertical and horizontal linkages between them. A credit risk shock in sector A will increase problem loans and defaults in that sector, but if sector B depends for business on sector A, problem loans and defaults might increase in sector B. Therefore, even in

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5 Indeed, banks are required to undertake such exercises under the terms of "Basel II".
6 Wittenberg (2001) reports Z-scores for unnamed business creditors in Israel. She had access to the proprietary data as an employee of the Bank of Israel.
7 Such as the Federal Reserve, the Bank of England, the Reserve Bank of Australia and the Bank of Italy. Our approach therefore has broad applicability.
the absence of counterparty risk, credit risk may be contagious. In principle, this mechanism may apply to individual firms. However, our data refer to sectors rather than firms. Nevertheless, if counterparty risk happens to be greater in sector A than sector B, the mean and the variance of credit risk should be higher in sector A than in sector B. We are unable to test the counterparty risk theory of contagion, but we are able to test the inter-industry trade theory of contagion.

Contagion and correlation may be understood in terms of the Reflection Problem due to Manski (1995) who distinguishes between correlated, contextual and endogenous effects. The former are induced by correlated unobservable shocks between sectors. Contextual effects arise when sectors are affected by common observable factors. For example, sectors A and B are both affected by the price of oil. Endogenous effects arise when there is a causal effect of A on B and/or vice-versa.

Endogenous effects might arise for a number of reasons related to the nature of the interaction between A and B. For example, A mimics B or B pressurizes A to conform. In epidemiology B contaminates A. Endogenous effects and contagion are synonymous. In practice it may be difficult to distinguish contagion from contextual and correlated effects.

The econometric identification of endogenous effects is problematic notwithstanding the specification of contextual and correlated effects. Identification of instantaneous causal effects between B and A requires instrumental variables that affect B but not A, and which affect A but not B. If, however, these effects are not instantaneous because contagion takes time, matters are simplified. To identify the causal effect of B on A we simply require that B’s outcome at time t-1 be weakly exogenous with respect to A’s outcome at time t.

In the case of infectious diseases contagion naturally takes time. Does financial contagion take time? Contagion induced by counterparty risk takes time because debtor A defaults after debtor B. So does contagion induced by inter-industry trade. Contagion in bank runs takes time because A runs after he sees B. If, however, a shock occurs which causes A and B to be mutually fearful of each other, then both A and B run at the same time. This behavior is empirically indistinguishable from correlated shocks.

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8 When Fitch downgraded US Treasury bonds in August 2011 there was no rational reason to sell these bonds, since the downgrade did not involve the release of new data. Had all investors been rational, we
Our focus on correlated credit risk may be regarded as an attempt to give empirical expression to the concept of systemic risk to which attention is being increasingly drawn. We decompose the correlation in bank credit risk for seven economic sectors into correlated, contextual and endogenous (contagious) effects. The former are captured by correlation between the residual errors in the empirical credit risk model. Two types of contextual effects are distinguished, macroeconomic and sectoral. Contagion is identified through the effect of lagged credit risk between sectors. Since the data are quarterly, we are forced to assume that contagion takes at least three months.

In summary, there are six main mechanisms which induce correlation in bank credit risk.

i) The business cycle and macroeconomic risk factors induce correlation across the economy as a whole.

ii) Sector-specific risk factors induce correlation between the sectors concerned.

iii) Correlation between sector-specific risk factors induce correlation between sectors.

iv) Unobservable shocks (represented by residuals) are correlated.

v) Counterparty risk induces correlation through contagion.

vi) Inter-industry trade induces correlation through contagion.

There is a further propagation mechanism, which does not concern us here. Chen (2001) notes that if credit risk triggers credit crunches (as in the Subprime Crisis), the contraction of liquidity may induce recession, which will induce further credit risk. Indeed, business cycles and credit cycles are interdependent (Kiyotaki and Moore 1997).

2 Factor Models

2.1 Macroeconomic and Sectoral Determinants of Credit Risk

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\(^9\) See, for example, Brunnermeier et al (2009).
Banks are assumed to have some control over their credit risk exposures by screening and monitoring their customers, especially in so-called “relationship banks”\textsuperscript{10}. They may also have preferences for customers in specific economic sectors where loan officers believe they possess proprietary information. The business environment in these economic sectors, which is beyond the control of banks, is hypothesized to affect customers’ ability to service debt. It may also be the case that adverse business environments induce banks to lower their lending standards, which in turn increases credit risk\textsuperscript{11}, because it is harder to find creditworthy customers in recessions.

We hypothesize a simple model in which loan officers rank loan applicants in a given sector (n) by their perceived credit risk. Loans are for standard amounts. Applicants’ are ranked in increasing order by their credit risk over the unit interval so that $s = 0$ for the best applicant and $s = 1$ for the worst. Applications with perceived credit risk below some cut-off $s^*$ are approved. The determination of $s^*$ is discussed below. Let $f(s)$ denote the density of credit risk. Credit risk exposure is therefore equal to $c = \int_0^{s^*} f(s)ds = F(s^*)$. In Figure 1 the horizontal axis measures bank credit ($s^*$) in sector n while the vertical measures credit risk ($c$) in sector n. Schedule A plots the relationship between credit and credit risk. It naturally slopes upwards, but the shape of schedule A depends on the density. Since credit risk cannot be negative the density cannot be normally distributed. The location of schedule A depends on exogenous factors which govern credit risk in the sector. If the business environment improves in the sector schedule A will shift downward or to the right. The opposite happens if the business environment deteriorates.

\textsuperscript{10} However, Dewenter and Hess (2003) do not find that credit risk is lower in relationship banking systems than in transaction banking systems.

\textsuperscript{11} As suggested by Bernanke and Gertler (1989), Rajan (1994) and Hellman et al (2000).
The credit risk policy of the bank is represented by schedule B, which hypothesizes an inverse relationship between credit risk and the amount the bank wishes to lend to sector n. Given everything else the bank wishes to allocate less credit to sector n the greater its credit risk relative to other sectors. Schedule B would be horizontal if the bank operated an absolute credit risk policy that required credit risk to be identical across sectors. Since the bank may have strategic reasons for lending to sector n we generalize by assuming that the bank operates a relative credit risk policy so that schedule B slopes downwards. The location of schedule B depends upon credit risk in rival sectors. Schedule B will shift to the right if credit risk in other sectors happens to increase. The equilibrium cut-off is determined at $E_0$ where schedules A and B intersect. Figure 1 shows that credit risk and exposure are jointly determined.

If the business environment deteriorates in sector n alone, schedule A will move leftwards to $A^1$ and the new equilibrium will be at $E^1$ at which credit in sector j is reduced and credit risk increases. If the business environment deteriorates outside sector n, schedule B will move rightwards to $B^1$ and the new equilibrium will be at $E^2$ at which credit and credit risk increase in sector n. If the business environment deteriorates symmetrically across all sectors, schedule A moves to the left and schedule B moves to the right. If banks are less choosy in recessions, as suggested by
Rajan (1994) and Hellman et al (2000), schedule B will move to the right such that credit risk increases, while credit remains largely unchanged. If, on the other hand, the recession induces a credit crunch schedule B might follow schedule A by moving leftwards; the banks reduce credit in all sectors to mitigate credit risk.

A fully structural model of credit risk would seek to estimate schedules A and B for all credit sectors. This would require instrumental variables to identify the two schedules. It would obviously be incorrect to estimate schedule A by regressing credit risk \( c \) on credit (\( s^* \)) since the latter is just as endogenous as the former. Reduced form estimation of credit risk regresses \( c \) on the drivers of schedules A and B. This is the approach which in fact we adopt in the present paper. We denote sectoral drivers by \( K \)-vector \( x \) and cyclical drivers by \( H \)-vector \( z \). Therefore \( c \) in sector \( n \) is hypothesized to depend on \( z \) and \( x \).

2.2 Contagion

Let \( y_t \) denote the vector of credit risk in sector \( n = 1,2,\ldots N \) at time \( t \). \( \Theta \) denotes the \( N \times N \) lattice of inter-sectoral credit relationships with elements \( \theta_{nj} \), which is zero along the leading diagonal and between immune sectors\(^{12} \). Since contagion may not be mutual \( \theta_{nj} \) may be zero when \( \theta_{jn} \) is positive. Contagion takes time, e.g. one period. We propose a simple first-order stochastic model of credit risk:

\[
y_t = \alpha + \Theta y_{t-1} + \Lambda y_{t-1} + Bx_t + \Phi z_t + u_t
\]

where \( B \) denotes the \( N \times K \) coefficient matrix of loadings with \( \beta_{nk} = 0 \) for factors that do not apply to sector \( n \), and \( \Phi \) denotes the \( N \times H \) matrix of factor loadings \( \phi_{nh} \). Whereas \( B \) is naturally a sparse matrix because \( x_k \) may only affect one or two sectors, \( \Phi \) is not sparse because most if not all credit risks depend on the macroeconomic factors. \( \Lambda \) is a diagonal matrix of inertial coefficients where \( \lambda_n \) is the autoregressive relationship between credit risk in sector \( n \) at time \( t \) and at time \( t-1 \). Finally, \( u \) is a vector of sectoral credit risk shocks, with variance-covariance matrix \( \Sigma_u \). If \( \Sigma_u \) is diagonal these shocks are independent across sectors. If it is not diagonal, this constitutes an additional source of credit risk correlation.

Because credit risk cannot be negative, equation (1) is in principle nonlinear. If \( y \) is a logistic function of credit risk (log odds default ratio) the non-negativity of credit risk is ensured.

\(^{12} \) We have assumed that \( \Theta \) is fixed. If firms reduce their business with risky counterparties, \( \Theta_t \) will be endogenous. We leave this difficult extension to future work.
The general solution to equation (1) is:

\[
y_t = (I_N - \Theta - \Lambda)^{-1} \alpha + \sum_{i=0}^{\infty} (\Theta + \Lambda)^i (Bx_{t-i} + \Phi z_{t-i} + u_{t-i}) + Ar'
\]  

(2)

where \( r \) denotes the vector of \( N \) eigenvalues of \( I_N - (\Theta + \Lambda)L \), \( A \) is the matrix of arbitrary constants obtained from the initial conditions, and \( L \) is the lag operator. Stationarity requires that these eigenvalues lie within the unit circle. Equation (2) defines the propagation mechanism of credit risk between sectors and over time. The impact multipliers are simply \( Bx_t + \Phi z_t + u_t \), but the higher order impulse responses depend on \( \Theta \) and \( \Lambda \). Contagion causes credit risk shocks to spillover onto other sectors since the coefficient matrix of \( u_t \) is \( (\Theta + \Lambda)^i \). In the absence of inertia \( (\Lambda = 0) \), this coefficient is simply \( \Theta^i \); it depends entirely on contagion. The same applies to sectoral and macroeconomic shocks.

From equation (1) the unconditional variance-covariance matrix of credit risk may be obtained \(^{13} \):

\[
\Sigma_y = E(yy') = \Sigma_{yx} + \Sigma_{yz} + \Sigma_{yu}
\]  

(3)

\[
\Sigma_{yx} = [I - (\Theta + \Lambda)]^{-1} B \Sigma_x B' [I - (\Theta' + \Lambda')]^{-1}
\]

\[
\Sigma_{yz} = [I - (\Theta + \Lambda)]^{-1} \Phi \Sigma_z \Phi' [I - (\Theta' + \Lambda')]^{-1}
\]

\[
\Sigma_{yu} = [I - (\Theta + \Lambda)]^{-1} \Sigma_u [I - (\Theta' + \Lambda')]^{-1}
\]

Equation (3) shows that the unconditional covariance matrix of credit risks may be decomposed into three components. The first (\( \Sigma_{yx} \)) is the factorial component induced by the covariance matrix of credit risk factors (\( \Sigma_x \)), which is assumed to be homoscedastic. The second (\( \Sigma_{yz} \)) is the cyclical or macroeconomic component where \( \Sigma_z \) denotes the covariance matrix for the macroeconomic variables. Finally, \( \Sigma_{yu} \) is the contribution of idiosyncratic credit risk shocks where \( \Sigma_u \) denotes the covariance matrix for \( u \), which will be diagonal if idiosyncratic credit risks are uncorrelated.

Having estimated \( \Theta, \Phi, \Lambda, B, \Sigma_x, \Sigma_z \) and \( \Sigma_u \) we may use equation (3) to decompose the covariance matrix of credit risk into its three component parts.

Since equation (3) is complex, we illustrate by assuming that there are two symmetric sectors \( A \) and \( B \) and one macroeconomic factor (\( z \)). Credit risk shocks have the same variance (\( \sigma^2 \)) in both sectors and their correlation coefficient is denoted by \( \rho \). Equation (1) therefore becomes:

\(^{13} \) For simplicity we ignore covariance terms between \( x \) and \( z \).
\[ Y_{.t} = \alpha_A + \theta Y_{.t-1} + \lambda Y_{.t-1} + \phi z_t + u_{.t} \]  
(4)

\[ Y_{.t} = \alpha_B + \theta Y_{.t-1} + \lambda Y_{.t-1} + \phi z_t + u_{.t} \]  
(5)

The variances and covariance\(^{14}\) for credit risk may be expressed in matrix form:

\[
\begin{bmatrix}
1 - \lambda^2 L & -\theta^2 L & -2\theta L \\
-\theta L & 1 - \lambda^2 L & -2\theta L \\
-\theta L & -\theta L & 1 - (\theta^2 + \lambda^2)L
\end{bmatrix}
\begin{bmatrix}
\text{var}(Y_{.A}) \\
\text{var}(Y_{.B}) \\
\text{cov}(Y_{.A}, Y_{.B})
\end{bmatrix}
= 
\begin{bmatrix}
\phi^2 \sigma_z^2 + \sigma^2 \\
\phi^2 \sigma_z^2 + \sigma^2 \\
\phi^2 \sigma_z^2 + \rho \sigma^2
\end{bmatrix}
\]  
(6)

The determinant of the coefficient matrix is:

\[ d = 1 + \pi_1 L + \pi_2 L^2 + \pi_3 L^3 \]

\[ \pi_1 = -(3\lambda^2 + \theta^2) \quad \pi_2 = 3\lambda^4 - \theta^4 + 2\lambda^2 \theta^2 \quad \pi_3 = (\theta^2 - \lambda^2)^2 \]

Solving equation (6) for the conditional covariance of credit risk, we obtain:

\[ \text{cov}(Y_{.A}, Y_{.B}) = \sum_{i=1}^{3} \pi_i \text{cov}(Y_{.A,t-i}, Y_{.B,t-i}) + (\rho \mu_1 + \mu_2) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2 + \sum_{n=1}^{3} A_n r_n^i \]  
(7)

\[ \mu_1 = (\theta^2 - \lambda^2)^2 - \theta^4 \quad \mu_2 = 2\theta \lambda (1 - \lambda^2 + \theta^2) \]

where \( r_n \) denotes the three eigenvalues of the coefficient matrix in equation (6) and \( A_n \) the associated arbitrary constants. The conditional covariance of credit risk is a third order autoregressive process, which converges to its unconditional counterpart:

\[ \text{cov}(Y_{.A}, Y_{.B}) = \frac{(\rho \mu_1 + \mu_2) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2}{1 + \pi_1 \pi_2 \pi_3} \]  
(8)

Solving equation (6) for the unconditional variance of credit risk in sectors A and B. we obtain:

\[ \text{var}(Y_t) = \sum_{i=1}^{3} \pi_i \text{var}(Y_{t-i}) + (\rho \mu_2 + \mu_1) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2 + \sum_{n=1}^{3} B_n r_n^i \]  
(9)

where \( B_n \) denote the arbitrary constants. The conditional variance of credit risk is also a third order autoregressive process, which converges to:

\[ \text{var}(Y) = \frac{(\rho \mu_2 + \mu_1) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2}{1 + \pi_1 \pi_2 \pi_3} \]  
(10)

Finally, dividing equation (8) by equation (10) gives the unconditional correlation of credit risk between the two sectors:

\[ r = \frac{(\rho \mu_1 + \mu_2) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2}{(\mu_1 + \rho \mu_2) \sigma^2 + \phi^2(\mu_1 + \mu_2) \sigma_z^2} \]  
(11)

\(^{14}\) The variances are calculated directly from equations (5A) and (5B). The covariance is obtained by multiplying equations (5A) and (5B).
Equation (11) shows that the correlation in credit risk depends on the structural parameters $\theta, \phi, \lambda, \rho, \sigma$, and $\sigma_z$. The correlation is zero when $\phi = \theta = \rho = 0$, in which event the variance of credit risk is $\sigma^2$ from equation (10) as expected. The following results are implied by equation (11):

i) Credit risk is more correlated the greater are $\phi$ and $\sigma_z$.

ii) If $\rho \mu_1 + \mu_2 < r(\mu_1 + \rho \mu_2)$ the correlation in credit risk varies inversely with $\sigma$.

iii) If $\mu_1 > r \mu_2$ the correlation in credit risk varies directly with $\rho$.

iv) Contagion increases the correlation in credit risk and its variance.

v) Inertia in credit risk increases the correlation in credit risk and its variance.

Case 1 in Table 1 serves as a baseline. Case 2 shows that contagion increases the correlation in credit risk and its variance. A comparison of cases 2 and 3 shows that inertia in credit risk increases the correlation and the variance. A comparison of cases 2 and 4 shows that the correlation varies inversely with the variance if credit risk shocks, while the variance obviously increases\(^{15}\). Cases 4 and 5 show, as expected, that the correlation varies directly with $\rho$, but so does the variance. In the absence of contagion and inertia (case 6) the correlation is 0.5 and the variance is 2 when credit risk shocks are uncorrelated. Finally, when credit risk is uncorrelated (case 7) the variance is 1.

### Table 1 Determinants of the Correlation in Credit Risk

<table>
<thead>
<tr>
<th>Case</th>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$\sigma^2$</th>
<th>$\phi$</th>
<th>$\rho$</th>
<th>$r$</th>
<th>Var(Y)</th>
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<tr>
<td>1</td>
<td>0.1</td>
<td>0.3</td>
<td>1.5</td>
<td>1</td>
<td>0.2</td>
<td>0.567</td>
<td>2.847</td>
</tr>
<tr>
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<td>0.2</td>
<td>0.3</td>
<td>1.5</td>
<td>1</td>
<td>0.2</td>
<td>0.614</td>
<td>3.065</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0</td>
<td>1.5</td>
<td>1</td>
<td>0.2</td>
<td>0.520</td>
<td>2.604</td>
</tr>
<tr>
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<td>0.2</td>
<td>0.3</td>
<td>1</td>
<td>1</td>
<td>0.2</td>
<td>0.681</td>
<td>2.478</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.597</td>
<td>2.446</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.500</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

$\sigma_z = 1$

### 2.3 Empirical Methodology

In view of the substantial heterogeneity between sectors, we do not treat our data as panel data. We therefore estimate individual models for each sector. This is feasible

\(^{15}\) This result is reversed if condition ii) does not hold.
because the data are available on a quarterly basis from 1997Q1 to 2010Q3. Let \( Y_{nt} \) represent an appropriate measure of bank credit risk in credit sector \( n \) at time \( t \), where the \( N \) sectors are defined in terms of different types of business (industry, services, persons etc). The \( z \)-factors are aliased by \( h = 1,2,\ldots,H \) and the \( x \)-factors are aliased by \( k = 1,2,\ldots,K \).

In sections 2.1 and 2.2 the dynamics were restricted to first order for expositional purposes. Inertia was first order, contagion occurred after one period, and the sectoral and macroeconomic risk factors affected credit risk instantaneously. In practice, inertia may be greater than first order, contagion might take longer than one period, and the risk factors might not affect credit risk instantaneously. Indeed, these dynamics have to be estimated from the data. We estimate the parameters of the model (\( \lambda \)'s, \( \beta \)'s, and \( \theta \)'s) using the following empirical model:

\[
Y_{nt} = \alpha_n + \sum_{i=1}^{p} \lambda_{ni} Y_{n-t-i} + \sum_{h=1}^{H} \sum_{i=0}^{b} \phi_{nh} z_{h-t-i} + \sum_{k=1}^{K} \sum_{i=0}^{d} \beta_{nk} x_{n-t-i} + \sum_{j=1}^{J} \sum_{i=1}^{c} \theta_{nj} Y_{j-t-i} + u_{nt} \tag{12}
\]

In equation (12) the \( \lambda \) coefficients capture inertia in bank credit risk, the \( \phi \) coefficients capture the dynamic effect of the systemic risk factors such as the business cycle, and the \( \beta \) coefficients capture the dynamic effects of the sectoral risk factors on bank credit risk. The \( \theta \) coefficients capture contagion effects. However, there may be negative contagion in which case \( \theta_{nj} < 0 \), i.e. an increase in credit risk in sector \( j \) reduces credit risk in sector \( n \). This will happen if credit risk in one sector is deflected to another, or if there is substitution in credit risk\(^{16}\). Finally, \( u_{nt} \) is a residual which may be correlated between sectors.

Equations (12) constitute a VAR-X model in credit risk in which the \( z \) and \( x \) risk factors are weakly exogenous and the shocks are correlated. This means that shocks to \( z \) and \( x \) propagate within and between sectors of the market for bank credit. \( X \) – shocks, which directly affect one sector will propagate onto other sectors via the \( \theta \) coefficients. \( Z \) - shocks, which directly affect more than one sector will propagate both within and between sectors inducing domino and boomerang contagion. Domino contagion occurs when credit risk in sector \( n \) spreads to sector \( j \) and thence to other sectors. Boomerang contagion occurs when credit risk in sector \( n \) rebounds back onto

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\(^{16}\) In epidemiological models exposure to a disease may strengthen immunity in which case \( \theta < 0 \).
sector n from sector j or third sectors. In Section 4 we simulate shocks to the z and x variables using the estimated model.

Identification of the model requires that $u_{nt}$ be serially independent within and between sectors otherwise the lagged dependent variables in equation (12) may not be weakly exogenous. Identification also requires that $z_t$ and $x_t$ be weakly exogenous, which requires that innovations in credit risk do not immediately affect the current state of the economy. If $z$ and $x$ are directly affected by credit risk they will not be weakly exogenous. If, however, credit risk has a lagged effect on $z$ and $x$, they may be weakly exogenous\textsuperscript{17}. Some of the risk factors are strongly exogenous because they refer to variables, such as the price of oil, which are determined abroad.

Estimation of the model proceeds as follows. Equation (12) is estimated by sector\textsuperscript{18} providing estimates of $\lambda$, $\beta$, $\phi$, and $\theta$. The estimates of $u_{nt}$ are then used to check for common unobserved factors. If these residuals are correlated between sectors and serially correlated we would use dynamic factor analysis (Stock and Watson 1988) to estimate the unobserved factors from the residuals. If instead the residuals are serially independent but correlated between sectors we would use static factor analysis to estimate the unobserved risk factors and their loadings. Finally, if the residuals are serially independent and are not correlated between sectors, there are no unobserved factors.

The lag lengths $p$, $b$, $c$ and $d$ are determined using the “general-to-specific” methodology of dynamic specification (Hendry 1995). Hypotheses about the risk factors are discussed in the next section. Misspecification checks are used to guard against the risk of data-mining. These include various LM tests as well as forecasting tests. The latter are particularly important since data-mined models typically forecast badly. We test the data for stationarity. If credit risk is trending it cannot be stationary. It might be argued that since credit risk is naturally bounded between zero and one, it must be inherently stationary. However, if credit risk is sufficiently persistent it may behave like a driftless random walk, in which event it is nonstationary\textsuperscript{19}. Indeed, in some sectors credit risk turns out to be nonstationary.

\textsuperscript{17} As e.g. in Chen (2001). When the subprime crisis struck in August 2007 it was not until the fourth quarter of 2007 that the US economy showed signs of recession.

\textsuperscript{18} We do not use panel data econometrics because we have sufficient time series observations (55) on each sector, and in any case the sectors are very heterogeneous.

\textsuperscript{19} The mean does not depend on time, but the variance does.
3 The Data

3.1 Defining Bank Credit Risk

Since 1997 the Supervisor of Banks at the Bank of Israel has published quarterly data on loan-loss provisions (write-offs) and problem credit for Israel's five main banking groups\(^{20}\). These data refer to total credit and do not unfortunately distinguish between different sectors of the credit market. However, the Bank of Israel also publishes data on loan-loss provisions and problem credit for different sectors of the credit market for the banking system as a whole. We use the latter data because we show that there is extensive heterogeneity in problem credit by sector than by banking group.

Unfortunately, we cannot estimate equation (12) for individual banks but only for the banking system as a whole.

Problem credit is defined by the Bank of Israel to include loans that are non-performing, are in temporary arrears, are under special supervision, are due to be rescheduled, or have been rescheduled. This definition of problem credit is broader than its counterpart at FDIC\(^{21}\), which consists of non-performing loans and impaired loans. The FDIC definition is roughly comparable to the first two components (non-performing and in temporary arrears). However, it is difficult to compare problem credit in Israel, where there is no swap market, with problem credit elsewhere because Israeli banks cannot off-load problem credit in the market for credit swaps. This means that problem credit in Israel may appear high because it remains on the balance sheet until it is written-off or ceases to be problematic. Our main measure of problem credit follows the Bank of Israel’s definition\(^{22}\).

An even narrower measure of problem credit would be loan-loss-provisions. These provisions are a formal accounting item that enters the profit and loss account of banks. Typically write-offs lag behind credit risk because banks only make loan-loss-provisions after it has become clear that the loan is beyond rehabilitation. And even then there may be accounting reasons when to declare the write-off.

Figure 2 Write-offs as a Percentage of Problem Credit

---

\(^{20}\) Bank Hapoalim, Bank Leumi, First International Bank, Discount Bank and United Mizrahi Bank. Smaller banks such as Bank Yahav are included in these groups.

\(^{21}\) As well as at leading rating companies and the IMF.

\(^{22}\) Since 2002 we have been able to compare the Bank of Israel's measure with its narrower FDIC counterpart. Although the former is naturally higher than the latter the correlation is quite high ($r = 0.92$).
In Figure 2 we plot the ratio of write-offs to problem credit for the Israeli banking system. The rate of loan-loss provisions out of problem credit typically varies between 1 percent and 3 percent per quarter (Figure 1) but it peaked at 5 percent in 2002. It is also seasonal; it is lowest in the first quarter and highest in the last, reflecting the fact that the tax year ends with the calendar year. There also seems to be a cyclical component to the rate of loan-loss provisions. There was a deep recession that began in the second half of 2000\textsuperscript{23}, reached its trough in 2002, and the economy began to recover in 2004. Incomplete data for 2007 – 2010 show that during the recession of 2008 the rate of write-offs increased but subsequently returned to 1 – 2 percent following the economic recovery. During the recession the rate of loan-loss provision was about 1 percentage point higher. One naturally expects loan-loss provisions to vary directly with problem credit, and they do. However, the timing of declaring loan-loss provisions seems rather haphazard\textsuperscript{24}.

3.2 Problem Credit

Table 2 The Sectoral Composition of Bank Credit and Credit Risk

<table>
<thead>
<tr>
<th>Industry</th>
<th>1997 Credit</th>
<th>1997 Problem Credit</th>
<th>2003 Credit</th>
<th>2003 Problem Credit</th>
<th>2010 Credit</th>
<th>2010 Problem Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>14.9</td>
<td>18.1</td>
<td>15.5</td>
<td>18.3</td>
<td>12.1</td>
<td>13.7</td>
</tr>
<tr>
<td>Building</td>
<td>20.8</td>
<td>25.9</td>
<td>13.3</td>
<td>29.4</td>
<td>11.7</td>
<td>33.1</td>
</tr>
<tr>
<td>Commerce</td>
<td>7.4</td>
<td>6.0</td>
<td>7.6</td>
<td>6.5</td>
<td>6.0</td>
<td>5.5</td>
</tr>
</tbody>
</table>

\textsuperscript{23} Due to collapse of the dotcom bubble and the outbreak of Intifada 2.

\textsuperscript{24} Hess (2007) too notes that in Australia write-offs are poorly correlated with problem loans.
Table 2 reports the sectoral composition of bank lending as measured by credit outstanding and the shares of these sectors in problem credit. The largest sectors are persons (including mortgages), building and industry. There have not been any major changes in the sectoral composition of bank credit except for financial services that have grown in importance. Some sectors such as industry and building are over-represented in problem credit, while others such as persons are under-represented. The substantial over-representation of agriculture in problem credit in 1997 resulted from the financial crisis in the kibbutzim and moshavim (agricultural cooperatives), which was subsequently solved through legislation and a political settlement.

We define the rate of problem credit (RPC) as the ratio of problem credit to the total amount of credit outstanding. RPC measures the ex-post probability that a shekel of bank credit is problematic. The first graph in Figure 3 plots RPC for all sectors. RPC fell from 10 percent in 1997 to 4 percent in 2010 and seems to be anticyclical. RPC fell during the dot.com boom at the end of the 1990s, increased during the recession of 2000 – 2004, fell during the subsequent economic recovery, increased during the recession of 2009, and fell with the economic recovery in 2010. However, the subsequent graphs, which plot RPCs for different sectors of the credit market, indicate a substantial degree of heterogeneity. For example, the last two graphs show that RPC for persons and business services has been falling continuously, while in other sectors, such as hostelry (tourism, hotels and restaurants) it has been increasing. In some sectors such as hostelry and building RPC is persistently high while in other sectors such as transportation and storage it is low.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hostelry</td>
<td>2.1</td>
<td>3.2</td>
<td>2.1</td>
<td>9.3</td>
<td>1.3</td>
<td>5.8</td>
</tr>
<tr>
<td>Transport &amp; Storage</td>
<td>2.3</td>
<td>1.3</td>
<td>2.5</td>
<td>0.9</td>
<td>2.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Communications &amp; Computer services</td>
<td>1.7</td>
<td>0.2</td>
<td>4.0</td>
<td>8.4</td>
<td>2.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Financial services</td>
<td>4.7</td>
<td>1.8</td>
<td>9.0</td>
<td>6.2</td>
<td>11.0</td>
<td>12.9</td>
</tr>
<tr>
<td>Business services</td>
<td>3.6</td>
<td>3.5</td>
<td>3.7</td>
<td>2.5</td>
<td>4.1</td>
<td>3.9</td>
</tr>
<tr>
<td>Public services</td>
<td>1.8</td>
<td>2.2</td>
<td>1.7</td>
<td>2.4</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Persons</td>
<td>27.9</td>
<td>15.8</td>
<td>30.3</td>
<td>13.4</td>
<td>38.7</td>
<td>14.9</td>
</tr>
<tr>
<td>Agriculture, electricity and water</td>
<td>12.8</td>
<td>21.8</td>
<td>10.2</td>
<td>2.6</td>
<td>9.4</td>
<td>1.6</td>
</tr>
</tbody>
</table>

**Figure 3 Rates of Problem Credit**
In some sectors such as industry and building RPC seems to follow the main trend, while in other sectors such as persons and business services RPC appears to buck the trend.

RPC is naturally bounded between zero and unity in which case its mean cannot increase or decrease without limit over time. The ADF and PP statistics in Table 3 indicate that RPC is nonstationary whereas the KPSS statistics indicate that in 5 out of 9 sectors we cannot reject the hypothesis that RPC is stationary. We therefore specify stationary factors in the five stationary sectors, and we specify nonstationary factors in the four nonstationary sectors, For all sectors we carry out a test for spurious regression as reported below.

**Table 3 Unit Root Tests for RPC**

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport &amp; Storage</th>
<th>Communications &amp; Computer services</th>
<th>Financial services</th>
<th>Business services</th>
<th>Persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-1.47</td>
<td>-1.22</td>
<td>-2.07</td>
<td>-1.90</td>
<td>-1.72</td>
<td>-1.67</td>
<td>-2.08</td>
<td>-3.67</td>
<td>0.36</td>
</tr>
<tr>
<td>PP</td>
<td>-1.28</td>
<td>-1.90</td>
<td>-1.42</td>
<td>-1.47</td>
<td>-1.55</td>
<td>-1.49</td>
<td>-1.90</td>
<td>-2.77</td>
<td>-1.27</td>
</tr>
<tr>
<td>KPSS</td>
<td>0.47</td>
<td>0.54</td>
<td>0.62</td>
<td>0.49</td>
<td>0.38</td>
<td>0.39</td>
<td>0.44</td>
<td>1.02</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Notes: ADF is the 4th order augmented Dickey-Fuller statistic and PP is the Phillips-Perron statistic with bandwidth 1-4 with critical value -2.9 at p = 0.05. KPSS is the 5th order KPSS statistic with critical value 0.463.

In Table 3 and the rest of the paper we drop sectors 9 (public services) and 11 (agriculture, water and electricity) of Table 2 since credit risk in these sectors has a political dimension.
The correlation matrix (Table 4) for sectoral RPCs contains negative as well as positive components, and suggests that there are substantial opportunities for hedging credit risk in a diversified loan portfolio. The correlations range between 0.9 and -0.38 and in only a few cases are the correlations close to zero.

Table 4 Correlation Matrix for the Rate of Problem Credit

<table>
<thead>
<tr>
<th>Industry</th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport &amp; Storage</th>
<th>Communications &amp; Computer services</th>
<th>Financial services</th>
<th>Business services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building</td>
<td></td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commerce</td>
<td>0.90</td>
<td></td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hostelry</td>
<td>0.17</td>
<td>0.89</td>
<td></td>
<td>0.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transport &amp; Storage</td>
<td>-0.16</td>
<td>0.00</td>
<td>-0.24</td>
<td>-0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Communications &amp; Computer services</td>
<td>0.05</td>
<td>0.74</td>
<td>0.13</td>
<td>0.89</td>
<td>-0.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial services</td>
<td>0.01</td>
<td>0.65</td>
<td>0.03</td>
<td>0.65</td>
<td>0.25</td>
<td>0.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business services</td>
<td>0.66</td>
<td>-0.27</td>
<td>0.74</td>
<td>-0.36</td>
<td>-0.15</td>
<td>-0.32</td>
<td>-0.32</td>
<td></td>
</tr>
<tr>
<td>Persons</td>
<td>0.70</td>
<td>-0.37</td>
<td>0.77</td>
<td>-0.31</td>
<td>-0.38</td>
<td>-0.24</td>
<td>-0.34</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Finally, Figure 4 plots RPCs for four banks. It shows that RPC varies by bank but typically falls in the range of 8 to 13 percent. In the late 1990s RPC for Bank Hapoalim, which is Israel’s largest bank, was relatively high, and RPC of the First International Bank of Israel (FIBI), founded in 1982, was relatively low. However, by 2002 FIBI had converged to the mean.

Figure 4 Rates of Problem Credit in Main Banks
4. Empirical Factor Models

Our main purpose in this section is to estimate equation (12) for seven main credit sectors, where Y is represented by the rate of problem credit (RPC). A large range of economy-wide risk factors (z) is hypothesized including, GDP and its components, unemployment, inflation, exchange rates, interest rates etc. We also use the Bank of Israel’s coincident indicator, which is intended to be correlated with the business cycle. We specify a broad range of sector-specific risk factors (x), which naturally vary between sectors. We use the general-to-specific methodology (Hendry 1995) to determine the dynamic structure of the individual factor models, and the choice of factors is largely determined by their ability to predict credit risk. To guard against over-fitting and data-mining, we apply a range of misspecification checks, including forecast tests and serial correlation tests. The models are estimated using quarterly data during 1997 Q1 to 2010 Q3 which covers the complete time span of the published data. We exclude from our analysis credit to the agricultural sector because, as mentioned, this sector has been the subject of legislation. We also exclude small and specialized sectors such as diamonds, electricity and water.

We have briefly mentioned the dilemma regarding the potentially nonstationary nature of the data on RPC during the sample period. In sectors where the rate of problem credit is trend-free, we use the Hodrick-Prescott filter to detrend factors, such as GDP, which have time trends. In sectors where the rate of problem credit is trending, we specify factors that are trending. The alternative would have been to specify credit risk factors that cointegrate with RPC for each of the sectors.

We check for spurious regression in two ways. First, we report unit root tests for the “long-run” residuals (u*) derived from the static counterparts of equation (12), i.e. when the lag structures are collapsed:

\[ u_{jt}^* = Y_{jt} - \alpha_j - \sum_{k=1}^{K} \beta_{kj} z_{it} - \sum_{n=1}^{N} \gamma_{nj}^* x_{nt} - \sum_{h \neq j}^{J} \theta_{hj}^* Y_{ht-1} \]  

\[ \beta_{kj}^* = \frac{\sum_{i=0}^{b} \beta_{ki}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \gamma_{nj}^* = \frac{\sum_{i=0}^{d} \gamma_{jni}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \quad \theta_{hj}^* = \frac{\sum_{i=1}^{e} \theta_{hji}}{1 - \sum_{i=1}^{p} \lambda_{ji}} \]

These long-run residuals should be stationary in the absence of spurious regression. Secondly, we carry out unit root tests on the full dynamic simulation (FDS) residuals, which substitute out the lagged contagion terms in equation (12) in terms of the z and
x variables which determine them. Therefore the second test takes account of contagion, whereas the first does not.

There are about 50 estimated model parameters. In the interests of digestibility we break down the presentation of these parameters as follows. In Tables 5 we report the factor models for each sector, i.e. the beta and gamma coefficients of equation (12). In Table 6 we report the estimated coefficients of inertia ($\lambda$) in equation (12) and in Table 7 we report estimated the contagion coefficients ($\theta$). Finally, diagnostic statistics are reported in Table 8. These include a Chow forecast test during 2007 Q1 – 2010 Q3, an LM test for upto 4th order autocorrelation within sectors, an LM test for 1st order correlation between sectors, and unit root tests for the long-run residuals and full dynamic simulation (FDS) residuals. Since some of the factors are specified in differences we use $d$ to denote the order of differencing and $s$ to denote the order of seasonal differencing.

**Table 5.1 Factor Models: Industry**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$d$</th>
<th>$s$</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial production$^a$</td>
<td>-6.76</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Share of electrical equipment in industrial production</td>
<td>-5.28</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

$^a$ Logged and HP filtered.

**Table 5.2 Factor Model: Building**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$d$</th>
<th>$s$</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction: gross output$^a$</td>
<td>-31.45</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Exchange rate$^b$</td>
<td>-9.87</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Consumption per head</td>
<td>-19.05</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Public sector investment</td>
<td>-5.90</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes. $b$: logs

**Table 5.3 Factor Model: Commerce**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$d$</th>
<th>$s$</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate (USD)</td>
<td>3.44</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
### Table 5.4 Factor Model: Hostelry

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign tourists(^b)</td>
<td>-3.25</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0.91</td>
</tr>
<tr>
<td>Domestic tourism(^b)</td>
<td>-25.86</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>6.17</td>
</tr>
<tr>
<td>Deaths due to terror</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>Real wages(^c)</td>
<td>-48.19</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>9.62</td>
</tr>
</tbody>
</table>

Notes: \(^b\) HP filtered.

### Table 5.5 Factor Model: Transport & Storage

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of diesel fuel</td>
<td>0.47</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>Employment in sector(^c)</td>
<td>-0.21</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>Wages in sector(^c)</td>
<td>0.00</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>Gross output in sector(^c)</td>
<td>-0.04</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>YTM indexed bonds</td>
<td>0.54</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.13</td>
</tr>
<tr>
<td>Exports</td>
<td>-3.41</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0.79</td>
</tr>
</tbody>
</table>

### Table 5.6 Factor Model: Financial Services

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment in sector(^a)</td>
<td>-25.69</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5.82</td>
</tr>
<tr>
<td>TASE 100(^c)</td>
<td>-2.57</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.52</td>
</tr>
</tbody>
</table>

### Table 5.7 Factor Model: Persons
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>d</th>
<th>s</th>
<th>Lag order</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate: Bank of Israel</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0.14</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.74</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes. The dependent variable is expressed as log\[RPC/(1-RPC)\].

The factors included in Tables 5 are diverse. In most sectors both sectoral and economy-wide risk factors are present. For example, Table 5.1 indicates that the rate of problem credit in industry varies inversely with the lag of the 2nd seasonal difference in industrial production and with the 4th lag of the share of electronic equipment in industrial production. Table 6 indicates that there is first order inertia (λ1 = 0.51) in the rate of problem credit in industry, and Table 7 indicates that there is positive contagion from credit risk in building, and negative contagion from the change in credit risk in transport and communications. Finally, Table 8 reports adjusted R² (0.95), the coefficient of variation of the residuals (6 percent), the p-value of the LM test statistic for upto 4th order autocorrelation in the residuals (not significant), and the p-value of a forecasting test of the model over the final 4 quarters of the sample (not significant). Finally, we report the ADF statistic for the long-run residuals (-6.75) and the KPSS statistic for the FDS residuals (0.11). These statistics do not suggest that the credit risk model for industry is spurious. We also tested for heteroscedasticity (White test) in the residuals and for upto 4th order ARCH in the residuals. In none of the sectors did these tests approach statistical significance. Therefore conditional bank credit risk is homoscedastic and does not display ARCH type heteroscedasticity.

Tables 5.2 through 5.7 report the risk factors estimated for the other credit sectors as well as their dynamic structure in terms of lags and levels versus differences. For example, credit risk in building varies inversely with building activity. It also varies inversely with the exchange rate since for most of the period houses were transacted in US dollars so that contractors benefited from devaluation. Some factors are clearly sectoral such as the price of diesel fuel in transport and arrivals of foreign tourists in the tourist sector. The macroeconomic factors include unemployment, inflation, consumption, investment, interest rates and the exchange...
rate. It should also be recalled that some sectoral factors are not independent of the business cycle (see below).

**Table 6 Coefficients of Inertia**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Persons</th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport-Storage</th>
<th>Financial services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag order</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes. P-value of coefficients of inertia < 0.025.

Table 6 shows that inertia in the rate of problem credit is pervasive and first order. Inertia is weakest in personal credit and strongest in building. Some of these coefficients of inertia appear close to 1, e.g. building (0.88). However, the ADF and KPSS statistics for building (Table 8) are -8.37 and 0.23 respectively, suggesting that the risk factors explain the behavior of credit risk over time, independently of inertia.

Table 7 shows that the rate of problem credit is also quite heterogeneous in terms of contagion. There are no immune sectors; all sectors are affected by contagion to a greater or lesser extent. All sectors are affected by contagion from at least two other sectors, and some are affected by three sectors. All sectors are contagious with the exception of the hotelry sector. Credit risk in hostelry does not affect other sectors, but it is affected by credit risk in building and among persons. Contagion is malignant in building amid among persons, but it is benign in commerce, industry and transport. This suggests that credit risk in the former sectors aggravates credit risk in other sectors, whereas credit risk in the latter sectors immunize other sectors against credit risk. They deflect credit risk away from these sectors.

Because there are no immune sectors and almost all sectors are contagious, boomerang contagion predominates over domino contagion. However, some sectors are more contagious than others. There are several aspects to this. First, the timing of contagion varies inversely with the lag orders in Table 7. For example, in the case of persons contagion is rapid (lag order 1) whereas in the case of building contagion is slower (upto lag order 4). Secondly, the size effects of contagion vary. Third, contagion is less persistent if it is in first differences than in levels. Fourth, sectors with more column entries in Table 7 are more contagious. For example, hostelry is not contagious at all, but credit risk in building affects all sectors apart from transport and storage.

**Table 7 Coefficients of Contagion**
Commerce | Industry | Transport Storage | Building | Hostelry | Financial Services | Persons
--- | --- | --- | --- | --- | --- | ---
Industry | -1.17 Δ(2) | 0.19 (1) | | | | |
Building | -0.68 Δ(3) | -0.45(4) | | | -0.32 Δ(1) | |
Commerce | | | 0.19 Δ(3) | | | |
Hostelry | | | | 0.56 (2) | 1.29 Δ(1) | |
Transport–Storage | -0.35 Δ(2) | | | | | |
Financial Services | | | -0.11 (3) | -0.18 Δ(3) | 0.07 (4) | |
Persons | | | | -0.06 (1) | -0.13 Δ(2) | 0.14 (2) |

Notes. The table reads horizontally. P-value of contagion coefficients < 0.025. ΔΔ indicates that contagion is in seasonal difference and (p) denotes that lag order of contagion.

<table>
<thead>
<tr>
<th>Table 8 Diagnostics</th>
<th>Adjusted R²</th>
<th>Standard error</th>
<th>CV</th>
<th>LM1</th>
<th>LM2</th>
<th>Forecast test</th>
<th>ADF long run residuals</th>
<th>KPSS dynamic simulation residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
<td>0.95</td>
<td>0.54</td>
<td>0.06</td>
<td>0.36</td>
<td>0.61</td>
<td>0.67</td>
<td>-6.75</td>
<td>0.11</td>
</tr>
<tr>
<td>Building</td>
<td>0.95</td>
<td>0.76</td>
<td>0.05</td>
<td>0.25</td>
<td>0.08</td>
<td>0.96</td>
<td>-8.37</td>
<td>0.23</td>
</tr>
<tr>
<td>Commerce</td>
<td>0.94</td>
<td>0.38</td>
<td>0.06</td>
<td>0.59</td>
<td>0.62</td>
<td>0.90</td>
<td>-7.58</td>
<td>0.15</td>
</tr>
<tr>
<td>Hostelry</td>
<td>0.98</td>
<td>1.31</td>
<td>0.05</td>
<td>0.32</td>
<td>0.60</td>
<td>0.36</td>
<td>-8.91</td>
<td>0.23</td>
</tr>
<tr>
<td>Transport-Storage</td>
<td>0.95</td>
<td>0.53</td>
<td>0.12</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
<td>-8.95</td>
<td>0.09</td>
</tr>
<tr>
<td>Financial services</td>
<td>0.92</td>
<td>0.48</td>
<td>0.14</td>
<td>0.30</td>
<td>0.22</td>
<td>0.40</td>
<td>-6.38</td>
<td>0.06</td>
</tr>
<tr>
<td>Persons</td>
<td>0.93</td>
<td>0.27</td>
<td>0.08</td>
<td>0.30</td>
<td>0.20</td>
<td>0.74</td>
<td>-9.99</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Notes. Dependent variable is RPC except for persons where it is log{RPC/(1-RPC)}. Standard error is measured in percentage points. LM1: p-value of F statistic of LM test for upto 4th order serial correlation in errors. LM2: p-value of F statistic of LM test for 1st order cross-autocorrelation. Forecast test: p-value of F statistic of Chow forecasting test for 2005 Q1 – 2010 Q3. CV is the coefficient of variation calculated at the mean of the data. ADF long run residuals is the Augmented Dickey-Fuller statistic for the residuals in equation (13). KPSS dynamic simulation residuals is the KPSS statistic for the residuals of a full dynamic simulation for 1998 – 2007.

Although R² exceeds 0.92, the accuracy of the models should be judged by their standard errors. For example, the standard error of 0.54 in the industrial sector should be compared to the rate of problem credit in the sector, which ranges between 6 -11 percent (Figure 2), and implies a coefficient of variation of 6 percent. Judging by the coefficient of variation, the model is least accurate in predicting the rate of problem credit in the financial services sector and most accurate in the building sector and in transport and storage.

The LM1 test indicates that in none of the sectors is there any evidence of serial correlation in the residuals, except perhaps in the case of personal credit. The LM2 test includes the lagged residuals for the other sectors in the auxiliary regression. A significant LM2 statistic would imply that the residuals are cross-autocorrelated.
with residuals in other sectors. The LM2 statistics are not statistically significant. Also, in none of the sectors does the model fail to predict the rates of problem credit except perhaps in transport and storage.

Since we are uncertain that RPC is stationary, we report ADF statistics for the long run residuals calculated using equation (13). These ADF statistics clearly show that the long run residuals are stationary\(26\), implying that the risk factors account for the trends in credit risk. Finally, we report the KPSS statistics for the residuals obtained from a full dynamic simulation of the model. This statistic takes into account that the contagion effects are jointly determined. In the absence of formal critical values, they suggest, on the whole, that the full dynamic simulation errors are stationary.

### Table 9 Correlation Matrix for Credit Risk Innovations

<table>
<thead>
<tr>
<th></th>
<th>Industry</th>
<th>Building</th>
<th>Commerce</th>
<th>Hostelry</th>
<th>Transport-Storage</th>
<th>Financial services</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Building</strong></td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Commerce</strong></td>
<td>0.347</td>
<td>0.021</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Hostelry</strong></td>
<td>0.208</td>
<td>0.030</td>
<td>0.092</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Transport-Storage</strong></td>
<td>0.174</td>
<td>0.184</td>
<td>0.036</td>
<td>0.187</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Financial services</strong></td>
<td>-0.053</td>
<td>0.079</td>
<td>-0.181</td>
<td>0.216</td>
<td>0.245</td>
<td></td>
</tr>
<tr>
<td><strong>Persons</strong></td>
<td>0.139</td>
<td>-0.165</td>
<td>0.025</td>
<td>-0.043</td>
<td>-0.290</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Notes. Bartlett test of sphericity = 26.9 P-value for zero correlations df = 21 is 0.172.

Finally, Table 9 reports the correlation matrix between the residuals of the sectoral models, i.e. it is the estimated correlation matrix for \(u_j\) in equation (12). Since the critical value of \(r\) at \(p = 0.05\) is 0.31 the majority of these correlations are not significantly different from zero. The correlations range between 0.347 and -0.29. Bartlett’s sphericity test indicates that the correlations in Table 9 are jointly not significantly different from zero. Therefore, credit risk innovations are mutually independent. Therefore despite the fact that according to Table 4 the rate of problem credit is correlated across sectors, the empirical model succeeds in orthogonalizing the residuals. We therefore rule out correlated innovations as a cause of correlation in credit risk.

### 5. Model Properties

\(26\) According to Mackinnon and Ericsson (2002) the critical values are about -3.7.
The empirical model consists of 7 dynamic equations, which are related through common factors, correlated factors and contagion. To investigate the properties of the model we first carry out a full dynamic simulation of the model, which solves for baseline solutions for the state variables (the 7 rates of problem credit) over the solution period (2000Q1 – 2006Q4). Next we shock the risk factors to generate new dynamic solutions. The difference between the perturbed and baseline solutions is the impulse response for the relevant shock. Since all shocks are temporary, we expect the impulse responses to die out over time unless the contagion coefficients are unstable. In the latter case, contagion induces an unstable domino-effect where credit risk in one sector destabilizes other sectors.

We begin by simulating the effect of a temporary increase of one percentage point in the rate of problem credit in the building sector, which is administered in the first quarter of 2000. The simulated impulse responses are reported in Figure 5. We naturally expect this shock to affect the rate of problem credit in the building sector but we are interested in how this shock spills-over to other sectors through domino and boomerang contagion. Table 7 shows that credit risk in building is particularly contagious; it affects credit risk in five other sectors. Therefore these sectors are directly affected in the simulation. Although credit risk in transport and storage is not directly affected, it is indirectly affected through the directly affected sectors. Furthermore, contagion boomerangs back onto credit risk in building. Figure 5 shows that although most of the propagation occurs within a year of the shock, it takes about three years for the effects of the shock to die out. The impulse responses indicate that the propagation of credit risk is a self-limiting process and that the epidemiology of credit risk is stable. They also show that in some cases, such as commerce, contagion may also induce overshooting.
Next, we administer a "cyclical shock", which has a pervasive effect on all sectors. Specifically, we administer a positive but temporary shock of one percentage point in the Bank of Israel's coincident indicator CI of economic activity in the first quarter of 2000. Although this indicator does not feature directly in Table 5 as a risk factor, many specified risk factors are correlated with it. These risk factors are regressed on CI to estimate their pro or anticyclical sensitivity. Therefore the shock to CI transmits itself to other macro risk factors, as well as to some of the sectoral risk factors. However, it does not transmit itself to sectoral risk factors such as foreign tourism or the price of diesel fuel. The impulse responses are plotted in Figure 6.

Figure 6 Cyclical Impulse Responses

The following regression was estimated for risk factor h: \( z_{ht} = a_h + b_h CI_t + d_h z_{ht-1} + e_h CI_{t-1} + v_{ht} \) where CI denotes the coincident indicator. The macroeconomic risk factors include, investment, consumption, the unemployment rate, real wages, exports and the TASE index. Note that some macroeconomic factors, such as the exchange rate, are acyclical. Note also that some of the sectoral factors are cyclical: turnover (building, transport and storage), industrial production, employment (financial services), real wages (transport and storage) and internal tourism.
Because the cyclical shock is positive, credit risk initially falls in all sectors except for building, where the drivers of credit risk are anti-cyclical. In some sectors, such as commerce, the fall is protracted and monotonic while in others, such as industry the fall is short-lived and non-monotonic. Indeed, in industry and hostelry credit risk increases after about a year and convergence is from above rather than below.

Finally, we decompose the standard deviation of credit risk into three components, macroeconomic (m), sectoral (s) and idiosyncratic (u). We illustrate using a simplified model in which credit risk is autoregressive but m, s and u are not:

\[ y_t = \lambda y_{t-1} + m_t + s_t + u_t \]  \hspace{1cm} (13)

The unconditional variance of \( y \) according to equation (13) is:

\[ \text{var}(y) = \frac{\text{var}(m) + \text{var}(s) + \text{var}(u) + 2 \text{cov}(ms) + 2\lambda[\text{cov}(m_y, y_{t-1}) + \text{cov}(s_y, y_{t-1})]}{1 - \lambda^2} \]  \hspace{1cm} (14)

Equation (14) uses the fact that by definition \( \text{cov}(um) = \text{cov}(us) = \text{cov}(uy_{t-1}) = 0 \).

To calculate the idiosyncratic component, \( \text{var}(u) \), we perturb the base-run for 2000 – 2010 by using the estimated residuals of the credit risk model (which are assumed to be zero in the base-run). The standard deviation of the difference between the base-
run and the simulation measures the contribution to the volatility in credit risk induced by idiosyncratic risk shocks (row 1 in Table 10). This simulation takes account of the dynamic effects of these shocks as they propagate through inertia within sectors and contagion between them. The contribution is largest in the hostelry sector and smallest in the household sector.

Row 2 in Table 10 reports the contributions of the macroeconomic risk factors to the volatility in credit risk. They are calculated by running a counterfactual simulation in which all the macroeconomic risk factors are held constant at their level in the first quarter of 2000. The standard deviation of the difference between the base-run and the perturbation is reported in row 2. The contribution of macroeconomic risk factors to the volatility in credit risk is also largest in hostelry and smallest among households.

The same applies to the contributions of sectoral risk factors to credit risk volatility, which are reported in row 3 of Table 10. They are calculated by running a counterfactual simulation in which all the sectoral risk factors are frozen at their level in the first quarter of 2000. The standard deviation of the difference between the base-run and the perturbation is reported in row 2.

We also carried out a counterfactual simulation in which CI is held fixed from 2000 Q1 and in which the cyclical macroeconomic and sectoral risk factors are dynamically correlated to CI. The difference between credit risk in the base-run and the counterfactual simulation measures the contribution of the business cycle to credit risk. Row 4 in Table 10 reports the contribution of the business cycle to the volatility in credit risk. Once again this contribution is largest in hostelry and smallest among households.

In row 5 of Table 10 we report the standard deviation of the data during 2000 – 2010. Notice that the squared standard deviations (variances) reported in rows 1-3 do not equal what is reported in row 5 because the decomposition needs to take into consideration the covariance between the macroeconomic and sectoral components. It also needs to take into consideration the covariances between m + s and lagged y. If m + s is independent of lagged y, the decompositions for financial services and households imply that the correlations between m and s are 0.071 and 0.008 respectively. For the other sectors the correlation exceeds 1 in absolute value, but this is because m + s is not independent of lagged y. This apparent inconsistency results from the fact that that the counterfactual simulations are carried out separately.
Nevertheless the decomposition in Table 10 sheds light on the relative contributions of the different types of risk to the volatility of credit risk within sectors. For example, in the industrial sector the major cause of credit risk is factorial whereas in the case of building the idiosyncratic component is as large as the sectoral component. In the hostelry sector the idiosyncratic component is very large because of the increase in the rate of problem credit that occurred after Intifada 2 (see Fig 3). Credit risk was most volatile in the hostelry sector (sd = 9.25%) because idiosyncratic and sectoral risk was very volatile. By contrast, household credit risk was least volatile because these components were small.

<table>
<thead>
<tr>
<th>Table 10 Decomposing the Standard Deviation of Credit Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1 Idiosyncratic</td>
</tr>
<tr>
<td>2 Macroeconomic</td>
</tr>
<tr>
<td>3 Sectoral</td>
</tr>
<tr>
<td>4 Cyclical</td>
</tr>
<tr>
<td>5 Credit risk</td>
</tr>
</tbody>
</table>

6. Conclusion

We have used aggregate bank credit risk data to estimate dynamic factor models in which the factors that determine credit risk are identified. These factors fall into two broad groups, macroeconomic factors that are related to the business cycle, and sector specific factors. There are three distinct dynamic aspects to the model. First, it takes time before factor shocks increase credit risk. Second, there is substantial inertia in credit risk so that it takes time for credit risk shocks to dissipate. Third, there is contagion of credit risk between sectors. In some cases, contagion is adverse so that credit risk in one sector induces credit risk in another. In other cases, however, contagion is benign so that some sectors benefit from credit risk elsewhere. Our model may be used to project credit risk by making assumptions about the various drivers of credit risk that feature in the model.

The model obviously cannot be used to estimate credit risk for individual creditors. However, we think that credit risk projections at the sectoral level may be used to estimate credit risk for individual creditors insofar as they operate in different sectors. Therefore if two otherwise identical creditors are from different sectors A and B, and the factor model predicts that credit risk is higher in sector A than in sector B,
the credit rating of the creditor in B should be higher than that of his counterpart in A. Banks will naturally try to reduce their exposure to sectors of the credit market that are projected to be riskier.

Indeed, as the recent subprime crisis has shown, credit risk may spread between different sectors of the credit market. A bearish outlook for house prices might have predicted the increase in credit risk not just in the market for mortgages, but also in other sectors of the credit market to which housing is related. We have shown that shocks in one part of the credit market spillover into other sectors via contagion.

Finally, in these troubled times of financial turmoil it is difficult not to comment on the lack of published data on banks' credit risk exposures. This paper could not have been written had the Bank of Israel not published the necessary data. However, like other regulatory bodies the Bank of Israel conceals most of the data. For example, it does not publish data on sectoral exposures of individual banks. Nor does the Federal Reserve or Bank of England publish detailed data on these exposures. Had such data been published, the public would have been able to make more informed judgments about the risk exposures of individual banks. One cannot help but wonder what might have transpired had the relevant regulatory bodies operated an enlightened policy of data transparency.
References


