Linking Large Currency Swings to Fundamentals' Shocks

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Abstract

The distribution of foreign exchange returns exhibits heavy tails. Under multiplicative parameter uncertainty, the distribution of macroeconomic fundamentals, like money supply and inflation, also exhibit heavy tails, while real income may not. In standard exchange rate models this translates into the tail behavior of exchange rate returns. To provide evidence for this extreme connection, we estimate the asymptotic dependence between the variables, rather than using regression analysis that focusses on average behavior. The strongest links are between Asian and Latin American currencies and fundamentals. The main drivers are monetary and financial variables; real income shocks appear disconnected.

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1 Introduction

It is well known that the distribution of foreign exchange (FX) returns exhibits heavy tails, to the extent that the probability of a large currency movement is of a different order of magnitude than the normal distribution would indicate. This fact has long been documented using specific distributions as in Westerfield (1977), Boothe and Glassman (1987) and Akgiray, Booth and Seifert (1988); or by zooming in on the tails as in Koedijk, Schafgans and de Vries (1990), Koedijk, Stork and de Vries (1992), Koedijk and Kool (1994) and Susmel (2001).[1]

Shocks that drive the fundamentals in standard theoretical exchange rate models also drive the exchange rate. Properties of the distribution of the fundamentals therefore determine the properties of the distribution of the exchange rate returns, apart from the influence of exogenous

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[1] The heavy tail feature partly stems from the other well documented FX feature that at higher frequencies exchange rate returns exhibit volatility clustering, see Diebold (1988). This adds to the fat tail nature of the return innovations, see Engle (1982), as the stationary distribution of an ARCH process exhibits heavy tails even if the innovations to the process are normally distributed.

noise. In this article we examine whether the well established heavy tail feature of the exchange rate returns can be attributed to the tail behavior of the macroeconomic fundamentals' distribution. We first explain why the distribution of some macroeconomic fundamentals can have heavy tails. Empirically, this feature is harder to observe for the fundamentals than for the exchange rate returns due to the low frequency nature of the fundamentals. Nevertheless, standard monetary macroeconomic models do have this theoretical implication. Subsequently, we show that in this case the exchange rate return and the macroeconomic fundamentals should exhibit asymptotic dependence[2]. This is similar to the volatility point made by Engel, Mark and West (2007): "Both sides should display comparable levels of volatility". We investigate whether this similarity also applies to the largest movements on both sides of the equation. In contrast, if for example the shocks of the fundamentals follow a multivariate normal distribution, though being correlated with the exchange rate returns, all dependency vanishes in the tail area. In this case the heavy tail feature of the exchange rate returns could solely derive from exogenous noise.

The literature on exchange rate modeling is burdened by the low explanatory power of macroeconomic fundamentals based specifications that rely on regression analysis. Forecasts based on such models have not fared well either. Short term no-change forecasts often produce a lower mean squared error, see Meese and Rogoff (1983) and Cheung, Chinn and Pascual (2005). Evidence in favor of the linkage between the nominal exchange rate and macroeconomic fundamentals is rather weak and often not robust, see Neely and Sarno (2002) and Sarno (2005). The intriguing exchange rate papers by Engel and West (2005) and Engel, Mark and West (2007), however, demonstrate that this poor performance of the fundamentals based regressions, known as the exchange rate disconnect puzzle, is due to the non-stationarity of the drivers and the high discount factor. Under these conditions, observed macroeconomic fundamentals can only weakly forecast exchange rate returns, even if the model is correct. The non-stationarity has been recognized for sometime. Evidence on a high discount factor is reported in Sarno and Sojli (2009).

The work by Engel and West (2005) indicates that lack of power to forecast does not necessarily imply the rejection of the exchange rate models, and hence the link between exchange rates and macroeconomic fundamentals. Consequently, there is a growing number of studies investigating the relationship between exchange rates and the fundamentals implied by the rational expectations present-value models of exchange rates. Using long run data, Balke, Ma and Wohar (2013) find that unobserved money demand shifts along with observed monetary fundamentals are an important contributor to the movements in the exchange rate. If exchange rates are driven by expected future fundamentals, as the present-value models suggest, then current exchange rates contain information regarding future fundamentals. Sarno and Schmeling (2014), indeed, show that in a large cross section of long-run data exchange rates have strong and significant predictive power for nominal fundamentals.

Predictability and the size of the regression coefficients are not the only ways in which the standard models can be judged. Engel, Mark and West (2007), for example, compare the volatility of exchange rate returns with the volatility of a constructed measure of the discounted expected future fundamentals. They find that the ratio of the two volatilities hovers between 30% and 50%, and conclude that fundamentals appear to be sufficiently volatile to match the FX volatility. In this article, we exploit the present-value exchange rate model of Engel and West (2005) to establish the linkage between the larger shocks that drive the observed fundamentals and the large movements of the exchange rate returns. To uncover the dependency between the exchange rate returns and the fundamentals in the tail areas of their distributions we rely on multivariate extreme value techniques from statistics. This is a method to identify whether larger movements

[2] The variables are said to be asymptotically dependent if the probability of a large realization of one variable, given a large realization of the other random variable, is non-zero, even in the limit.

of the economic fundamentals are associated with large currency swings. It may give different information than regression based techniques that elicit average behavior, since the focus is exclusively on the tail areas.

It is well documented that exchange rate returns exhibit heavy tails. Not much is known about the fundamentals in this respect. This is, at least in part, due to the low frequency nature of these data. We first argue theoretically that within a standard monetary macroeconomic model with multiplicative parameter uncertainty, the implied distributions of economic variables like the interest rate, inflation rate and money stock can exhibit the heavy tail feature, even if the multiplicative noise distributions have no tails at all (such as the uniform distribution). It also follows from the model that the distribution of real income shocks, per contrast, may have thin tails. Subsequently, the additive property of heavy-tailed distributions is used to show that this property carries over to the composite fundamentals. This implies that if the fundamentals exhibit heavy tails, then standard type FX models induce these heavy tails onto the exchange rate returns. But not only that, those models also imply the specific asymptotic dependency, through the linearity of the exchange rate model.

Using monthly observations from 34 countries over the period February 1974 to May 2016, these implications are investigated separately for left and right tails. We do indeed find that financial and monetary variable shocks appear strongly connected with the larger movements in the exchange rates on the depreciation side, but industrial production is not. This accords with theoretical model as well as the findings in Balke, Ma and Wohar (2013) and Sarno and Schmeling (2014). The intensity of the interdependency in the tail area varies across regions. The intensity is generally higher for Latin American and Asian countries.

2 Theory

Within a standard monetary macroeconomic model we first show that multiplicative parameter uncertainty with a bounded support induces heavy tails on the distribution of the macroeconomic aggregates, even in the setup that the noise itself does not have heavy tails. Then, we review the present-value exchange rate model of Engel and West (2005) to establish the tail linkage between the observed fundamentals and the exchange rate returns. Subsequently, we provide a short review of the probabilistic properties of fat-tailed distributed random variables and the strong linkage that this may imply for macroeconomic shocks and exchange rate returns.

2.1 Tail Events and Macroeconomic Fundamentals

One may wonder why macroeconomic fundamentals have distributions with heavy tails. An early statistically oriented explanation for inflation rates was offered by Engle (1982). Engle's ARCH model has random variables follow a martingale process with autoregressive behavior in the second moment, causing clusters of high and low volatility. Even if the innovations are thin-tailed normally distributed, the stationary distribution ends up having heavy tails like the Pareto distribution, see de Haan, Resnick, Rootzen and de Vries (1989). In the vein of Bollerslev (1987), Cumperayot (2002) shows that macroeconomic variables still exhibit heavy tails after filtering out the ARMA-GARCH components.

Here we develop an economic based explanation of how the distribution of macroeconomic variables like the money stock, interest rate or inflation rate can exhibit the heavy tail feature. The idea is not to present a fully fledged theory, as this would be outside the scope of this article, but to present a coherent argument for the macroeconomic variables involved. The next subsections then show how the heavy tail feature is carried over to the exchange rate returns.

To this end consider the following standard comprehensive monetary macroeconomic model, as presented in Walsh (2003, p.440) with Brainard (1967) type uncertainty. The aggregate supply curve reads

$$Y_t = A_t(\Pi_t - E_{t-1}[\Pi_t]) + \varphi_t, \tag{1}$$

where Y_t is the logarithmic level of output, Π_t is the inflation rate and $E_{t-1}[\Pi_t]$ is the time $t-1$ expected inflation for time t, and φ_t is a noise term. In the short run, deviations from the long-run output level are possible due to expectational errors. The elasticity of output with respect to inflation expectations' errors is A_t. Below we discuss the variability of A_t. Thus, equation (1) is in a crude way the Lucas type supply curve amended with Brainard type multiplicative uncertainty.

Aggregate demand depends on real interest rates, i.e. the nominal interest rate minus expected inflation $I_t - E_{t-1}[\Pi_t]$:

$$Y_t = -b(I_t - E_{t-1}[\Pi_t]) + \eta_t. \tag{2}$$

The money market demand equation is based on the quantity equation

$$M_t = P_{t-1} + \Pi_t + \gamma Y_t - \lambda I_t + v_t, \tag{3}$$

where M_t and P_{t-1} stand for the logarithms of the quantity of money and the price level, respectively; and v_t is the velocity shock. The λ is the semi-interest rate elasticity of money demand. A logarithmic expansion of the money supply gives

$$M_t = mm + MB_t, \tag{4}$$

where mm and MB_t are the logarithms of a constant money multiplier and the monetary base, respectively. The money multiplier is well known not to be constant, but in the above setup, this is captured by the velocity shocks.

The additive demand and supply shocks (φ_t, η_t) are assumed to exhibit mean zero i.i.d. noise that follows a distribution with thin (exponential declining) or bounded tails (in case of bounded support). At this point we do not need to be specific about the shocks v_t to the money market equation. The v_t shocks are not required to be independent over time, i.e. these may follow a stochastic process.[3]

The central bank's objective function is assumed as in Calvo and Reinhart (2002). In logarithmic form, the central bank maximizes the expected welfare function

$$\max_{I_t} E_{t-1}[W_t] = \omega_{t-1} E_{t-1}[MB_t - P_t + I_t] - \frac{1}{2}(1-\omega_{t-1}) E_{t-1}\left[(\Pi_t - \pi^*)^2\right], \tag{5}$$

where the weight ω_{t-1}, reflects the degree of trade-off between the seigniorage from the central bank's money creation and the goal to stabilize the inflation rate around its target π^*. The weight ω_{t-1} ranges between 0 to 1. At $\omega_{t-1} = 0$, all emphasis is on containing inflation, while for $\omega_{t-1} = 1$ seigniorage collection is the main goal. Over time and across countries ω_{t-1} varies.

According to Calvo and Reinhart (2002), attempting to exploit a Phillips curve by monetary authorities is of little practical relevance for most emerging markets. Instead, in many emerging economies inflation surprises are used to generate additional revenue from money creation and to reduce the real value of nominal government debt and public sector wages. This feature generally differentiates the emerging market economies from, e.g., the eurozone countries with the European Central Bank's single inflation stability objective in which case $\omega_{t-1} = 0$.

[3] Walsh (2003) solves the model under the assumption that the shocks follow an AR(1) process.

Based on information available at time $t-1$, the central bank determines the policy interest rate I_t in order to maximize expected welfare. To find the optimal policy interest rate, we first eliminate Y_t from the first two equations (1) and (2) to get

$$\Pi_t = -\frac{b}{A_t} I_t + \left(1 + \frac{b}{A_t}\right) E_{t-1}[\Pi_t] + \frac{\eta_t - \varphi_t}{A_t}.$$

By taking expectations conditional on time $t-1$ information,

$$I_t = E_{t-1}[\Pi_t].$$

From equation (2), it implies that

$$Y_t = \eta_t. \tag{6}$$

The domestic inflation rate then follows as

$$\Pi_t = I_t + \frac{\eta_t - \varphi_t}{A_t}.$$

To derive the seigniorage term in (5), we use equations (2), (3) and (4)

$$MB_t - P_t + I_t = (1-\lambda) I_t + v_t - mm + \gamma\eta_t.$$

Using the expression for Π_t in terms of I_t, the central bank's expected welfare function can be written as

$$\max_{I_t} E_{t-1}[W_t]$$

$$\begin{aligned} = \; & \omega_{t-1} E_{t-1}[(1-\lambda) I_t - mm] - \frac{1}{2}(1-\omega_{t-1}) E_{t-1}\left[(I_t - \pi^*)^2\right] \\ & -\frac{1}{2}(1-\omega_{t-1}) E_{t-1}\left[\left(\frac{\eta_t - \varphi_t}{A_t}\right)^2\right]. \end{aligned}$$

The optimal interest rate policy follows as

$$I_t = \pi^* + (1-\lambda)\frac{\omega_{t-1}}{1-\omega_{t-1}}. \tag{7}$$

For the single price stability objective, i.e. when $\omega_{t-1} = 0$, $I_t = \pi^*$. However, for countries which prefer some seigniorage from money creation, a higher interest rate, induced by higher expected inflation, implies higher expected welfare.

Solving for the inflation rate, we find

$$\Pi_t = \pi^* + (1-\lambda)\frac{\omega_{t-1}}{1-\omega_{t-1}} + \frac{\eta_t - \varphi_t}{A_t}. \tag{8}$$

The money market equation (3) implies that

$$M_t = P_{t-1} + (1-\lambda)\pi^* + (1-\lambda)^2 \frac{\omega_{t-1}}{1-\omega_{t-1}} + \frac{\eta_t - \varphi_t}{A_t} + \gamma\eta_t + v_t. \tag{9}$$

To explain the heavy tail feature of macroeconomic fundamentals, we rely on two assumptions of multiplicative parameter uncertainty: Uncertainty regarding the structural parameters and uncertainty about the central bank's preferences. Since the seminal work of Brainard (1967), the topic of policy effectiveness has received considerable attention. Frequently model estimates and new data lead to parameter revisions, see Sack (2000). We capture the model uncertainty by assuming that the coefficient for the short-run Phillips effect A_t is an i.i.d. random variable with positive mean. Swamy and Tavlas (2007) offer theoretical arguments in favor of a random coefficient specification, while Vavra (2014) offers empirical support for the time-varying slope of the Phillips curve. The variability of the slope A_t in equation (1) captures changes in wage indexation to inflation. Over time and across countries, indexation has varied considerably. The level of indexation determines the responsiveness of output to inflation. Full indexation implies $A = 0$. This corresponds to the classical dichotomy. In case of partial indexation in the short run, output responds to changes in inflation.

Changes in the relative weight ω_{t-1} that the central bank assigns to its objectives is our second source of parameter uncertainty in the model that may be the cause for heavy tails. Levin and Williams (2003) show how weights in the central bank's objective function are affected by changes in the economic structure.[4] For empirical evidence, Surico (2007) studies the asymmetric preferences of the Fed. He shows that the central banks may put different weights on positive and negative deviations of inflation, output and the interest rate from their reference values. The weight changes through time. Assenmacher-Wesche (2006), in addition, indicates that the central bank policy in the US, UK and Germany differs across low and high inflation regimes. Weights in the central banks' objective functions thus may alter over time and certainly vary across countries.

To analyze the implication of parameter uncertainty on the tails of macroeconomic variables, we assume that both the random coefficient A_t in the Phillips curve and the weight component $(1-\omega_{t-1})$ assigned to the central bank's inflation objective follow beta distributions of the form

$$\Pr\{X \leq x\} = x^{\alpha}, \;\; \alpha > 2. \tag{10}$$

The support of this distribution is $[0, 1]$.[5] The beta distribution is clearly not fat tailed. However, the presence of the terms $(\eta_t - \varphi_t)/A_t$ and $\omega_{t-1}/(1-\omega_{t-1})$ results in heavy-tailed distributions of the interest rate I, inflation rate Π and money supply M.[6] The tail feature of real income Y, however, exclusively depends on the thin-tailed distribution assumed for η.[7]

Consider the term $(\eta_t - \varphi_t)/A_t$. Suppose that A_t has a beta distribution (10)

$$\Pr\{A \leq x\} = x^{\alpha_A}, \;\; \alpha_A > 2.$$

The distribution of the inverse of A is

$$\Pr\{\frac{1}{A} \leq x\} = 1 - \Pr\{A \leq \frac{1}{x}\} = 1 - \frac{1}{x^{\alpha_A}}, \tag{11}$$

with support $x\epsilon[1, \infty)$. Thus the inverse of A has a heavy-tailed Pareto distribution and has moments k only up to α_A.

As a result, the unconditional distribution of $(\eta_t - \varphi_t)/A_t$ is also heavy tailed. To see this, let $Q = \eta - \varphi$ and consider the distribution of Q/A. Suppose the distribution of Q is such that

[4] Furthermore, Nobay and Peel (2003) investigate the implications for optimal monetary policy in a model of asymmetric central bank preferences. Walsh (2005) examines the impacts of the central bank's endogenous objective function on the evaluation of targeting rules for monetary policy.

[5] The fact that zero is in the support of A reflects the possibility that the short-run supply curve may be vertical, i.e. it coincides with the long-run supply curve. For $1-\omega$, a zero means that the country ignores the price stability objective.

[6] By means of Feller's Convolution Theorem (1971, VIII.8), it is easy to show that if the fundamentals like money supply or interest rates have heavy tails, their time differentials are also heavy tailed.

[7] The results below may appear specifically due to the assumption of the beta distribution in (10). What is crucial is that zero is in the support of the distribution. The result therefore also follows if we had assumed an exponential distribution, say.

at least the α-th moment is bounded; thus $E_Q[Q^\alpha] < \infty$. Using the conditioning argument of Breiman and the argument in (11) then shows that

$$\begin{aligned}\Pr\{\frac{Q}{A} &> x\} = E_Q[\Pr\{\frac{q}{A} > x|Q=q\}] \\ &= E_Q[\Pr\{\frac{1}{A} > \frac{x}{q}|Q=q\}] \\ &= E_Q[\left(\frac{Q}{x}\right)^{\alpha_A}] = E_Q[Q^{\alpha_A}]x^{-\alpha_A}. \end{aligned} \tag{12}$$

Note that the numerator only plays a minor role, since all that is required is that the expectation $E_Q[Q^{\alpha_A}]$ exists. Therefore, due to the random Phillips curve coefficient A in the denominators in equations (8) and (9), the unconditional distributions of Π and M have heavy tails.

For the relative weight term $\omega_{t-1}/\left(1-\omega_{t-1}\right)$, assume that $(1-\omega_{t-1})$ follows a beta distribution (10)

$$\Pr\{1-\omega \leq x\} = x^{\alpha_\omega}, \;\; \alpha_\omega > 2.$$

The distribution of the relative weight $\omega_{t-1}/\left(1-\omega_{t-1}\right)$ follows as

$$\begin{aligned}\Pr\{\frac{\omega}{1-\omega} &\leq x\} = \Pr\{1+\frac{\omega}{1-\omega} \leq 1+x\} = \Pr\{\frac{1}{1-\omega} \leq 1+x\} \\ &= 1-\Pr\{1-\omega \leq \frac{1}{1+x}\} = 1-\left(\frac{1}{1+x}\right)^{\alpha_\omega} \\ &= 1-x^{-\alpha_\omega}\left(\frac{1}{1/x+1}\right)^{\alpha_\omega}.\end{aligned}$$

The relative weight has a Burr distribution on $[0,\infty)$ with a tail in which the Pareto term $x^{-\alpha_\omega}$ dominates as $x \to \infty$ (since $1/x+1$ tends to 1). The expressions (7), (8) and (9) for the macroeconomic variables I, Π and M contain $\omega_{t-1}/\left(1-\omega_{t-1}\right)$. Thus the policy weights provide a second possible source for heavy-tailed distributed macroeconomic variables. Nevertheless, the real income Y is not heavy tailed, unless the distribution of η is heavy tailed itself.

The multiplicative parameter uncertainty explains why macroeconomic fundamentals may be heavy-tailed distributed. Yet, one might still ask, why do typical macroeconomic models not display heavy tails in the solution of the models? Given the complexity of a typical micro-based macro model, the solution often entails calibration after linearizing the model around the non-stochastic steady state. Consider the effects of such linearization for two different cases, one involving a product of two random variables, SR say, and one involving a ratio Q/A. For the product case, a second order Taylor approximation does a perfect job, since at $S=s$, $R=r$

$$SR|_{s,r} = sr + r(S-s) + s(R-r) + (S-s)(R-r) = SR.$$

Next, consider the second order Taylor expansion for the ratio around $Q=q$ and $A=a$

$$\begin{aligned}\frac{Q}{A}|_{q,a} &= \frac{q}{a}+\frac{1}{a}\left(Q-q\right)-\frac{q}{a^2}\left(A-a\right)+\frac{1}{2}2\frac{q}{a^3}\left(A-a\right)^2-\frac{1}{a^2}\left(Q-q\right)\left(A-a\right) \\ &= \frac{q}{a}+2\frac{1}{a}Q-2\frac{q}{a^2}A+\frac{q}{a^3}A^2-\frac{1}{a^2}QA.\end{aligned}$$

The random ratio never shows up again, no matter how many terms in the Taylor approximation are used. Around a particular point the ratio can be approximated as precise as is desired, but the global stochastic property of heavy tails stemming from the ratio is lost. This is particularly relevant in the case of multiplicative Brainard type uncertainty.

2.2 The Canonical Exchange Rate Model

To establish the link between the extreme exchange rate movements and the observed macroeconomic fundamentals, we rely on the present-value exchange rate model of Engel and West (2005). Below, we briefly recap their model and subsequently discuss limit copula, a concept we use to assess the tail dependency of multivariate probability distributions.

From equation (3), we have

$$M_t = P_t + \gamma Y_t - \lambda I_t + v_t.$$

Assume that a similar relation holds abroad. Then, by taking the difference of the two expressions yields the quantity equation of money at home relative to a base country

$$m_t = p_t + \gamma y_t - \lambda i_t + \nu_t,$$

where lower case letters denote relative country variables and ν stands for the difference between the country specific velocity shocks.

Let s be the logarithm of a nominal exchange rate, quoted per one unit of the base country's currency in terms of the domestic currency. The real exchange rate z equals the nominal exchange rate s minus the relative prices, i.e. $z = s - p$. Substituting this into the above equation yields

$$s_t = m_t + z_t - \gamma y_t + \lambda i_t - \nu_t.$$

Furthermore, let ρ be the deviation from uncovered interest parity (UIP), so that (recall that i is the interest rate differential)

$$i_t = E_t\left[s_{t+1}\right] - s_t + \rho_t.$$

Using the UIP relation in the expression for the exchange rate gives the monetary approach based Cagan style exchange rate expression

$$s_t = \frac{1}{1+\lambda}\left[m_t + z_t - \gamma y_t - \nu_t\right] + \frac{\lambda}{1+\lambda}\rho_t + \frac{\lambda}{1+\lambda}E_t\left[s_{t+1}\right]. \tag{13}$$

Use the following shorthand notation

$$x_t = m_t + z_t - \gamma y_t - \nu_t \tag{14}$$

for the fundamentals, including the country relative velocity shock. Forward iteration yields the standard no-bubbles solution to (13)

$$\begin{aligned} s_t &= \frac{1}{1+\lambda}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j E_t\left[x_{t+j}\right] \qquad (15) \\ &\quad + \frac{\lambda}{1+\lambda}\sum_{j=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^j E_t\left[\rho_{t+j}\right]. \end{aligned}$$

To fix ideas, it helps to consider specific stochastic processes for the fundamentals x and the deviation from UIP ρ. Suppose as in Engel, Mark and West (2007) that the fundamentals have a unit root and that the changes in the fundamentals follow a stationary $AR(1)$ process[8]:

$$\Delta x_t = \phi \Delta x_{t-1} + \varepsilon_t,\ E_t\left[\varepsilon_{t+1}\right] = 0,\ \phi \epsilon (0,1). \tag{16}$$

[8] For this assumption to be consistent with the macroeconomic model one requires that the first differences in the velocity shocks $\Delta\nu_t$ also follow this $AR(1)$ process with parameter ϕ.

Here Δ is the difference operator and ε represents i.i.d. shocks that drive the composite fundamentals.

Regarding the deviations from UIP, we assume an $AR(1)$ process driven by risk factors μ. The risk factors may comprise some of the fundamentals from x and other risk drivers, see Burnside, Eichenbaum, Kleshchelski and Rebelo (2011) and Sarno, Schneider and Wagner (2012). Here we follow Engel and West (2005) and do not explicitly state what these factors are. But we do allow that ε and μ are possibly correlated. Furthermore, we do assume that the deviations from UIP are stationary. This is also the preferred option by Engel and West (2005, p. 496), but given the presumed large $AR(1)$ coefficient, they use the $I(1)$ specification as a shortcut. Our specification for the deviations from UIP thus reads

$$\rho_t = a\rho_{t-1} + \mu_t,\ E_t\left[\mu_{t+1}\right] = 0,\ a\epsilon(0,1) \tag{17}$$

where the shocks μ have zero mean and possibly the covariance $\sigma_{\varepsilon,\mu} \neq 0$. While the two may be interdependent at any instant, over time the two shocks are assumed to be i.i.d.

Given the stochastic process assumptions for the fundamentals (16), it follows that

$$(1-b)\sum_{j=0}^{\infty} b^j E_t\left[x_{t+j}\right] = x_t + \frac{b\phi}{1-b\phi}\Delta x_t,$$

where the discount factor b is shorthand for

$$b = \frac{\lambda}{1+\lambda}.$$

The UIP expression (17) implies

$$b\sum_{j=0}^{\infty} b^j E_t\left[\rho_{t+j}\right] = \frac{b}{1-ab}\rho_t.$$

Combine these expressions in equation (15) and lag the growth rate of fundamentals to obtain the following expression for s_t in terms of observable fundamentals, UIP deviation and current shocks

$$s_t = x_t + \frac{b\phi^2}{1-b\phi}\Delta x_{t-1} + \frac{b}{1-ab}\rho_t + \frac{b\phi}{1-b\phi}\varepsilon_t. \tag{18}$$

By taking first differences of (18) yields the expression for the exchange rate returns

$$\Delta s_t = (1-b)\frac{\phi}{1-b\phi}\Delta x_{t-1} - \frac{(1-a)\,b}{1-ab}\rho_{t-1} + \frac{b}{1-ab}\mu_t + \frac{1}{1-b\phi}\varepsilon_t. \tag{19}$$

The expression (19) gives the coefficients one would find in a regression of the exchange rate returns on the growth rates of the lagged fundamentals. Engel and West (2005) and Engel, Mark and West (2007) argue and demonstrate that typical values of the discount factor b are close to 1, while Sarno and Sojli (2009) provide empirical evidence. By equation (19) this implies that the changes in the fundamentals become unimportant relative to their shocks ε_t, the lagged deviation from UIP ρ_{t-1} and the exogenous UIP shocks μ_t. Furthermore, the exchange rate can have the appearance of a random walk if ρ is small or highly persistent. As a result, beating a random walk in forecasting exchange rates is too strong a criterion for judging an exchange rate model (Engel, Mark and West, 2007).

In this article, we propose the limit copula (*or tail analysis*) as an alternative for evaluating the association between the exchange rate returns and the lagged fundamentals, as predicted by the model in equation (19). The limit copula focusses specifically on the dependency in the tails. The objective is to see whether large movements of macroeconomic fundamentals are associated with extreme movements of exchange rates.

One interpretation of regression coefficients is in terms of second moments:

$$\frac{E[XY]}{E[X^2]} = \frac{\int\int xyf(x,y)dxdy}{\int x^2 f(x)dx}.$$

Given the particulars of the exchange rate model in (19), this average response is small. As an alternative to the regression analysis, one can uncover the influence of the larger movements in Δx_{t-1} on Δs_t by using the limit copula and treating positive and negative shocks separately. Averaging across quadrants might make the effects disappear.

So, instead of using the first and second cross moments to elicit the dependency, we use probabilities in a particular quadrant, which are the corresponding zero moments:

$$\frac{\int_q \int_q x^0 y^0 f(x,y)dxdy}{\int_q x^0 f(x)dx} = \frac{\Pr\{X > q, Y > q\}}{\Pr\{X > q\}}.$$

If the large macroeconomic fundamental shocks are the primary drivers of the largest exchange rate changes, we can zoom in on the tail areas. This gives the so called limit copula

$$\lim_{q\to\infty} \frac{\Pr\{X > q, Y > q\}}{\Pr\{X > q\}} = \lim_{q\to\infty} \frac{\int_q \int_q x^0 y^0 f(x,y)dxdy}{\int_q x^0 f(x)dx}, \tag{20}$$

see McNeil Frey and Embrechts (2005, Ch. 5.2). It indicates the probability that the random variable Y is above the threshold q, conditional on the random variable X being above q, as the threshold q approaches infinity. For joint distributions, like the multivariate normal distribution this limit is zero; but, there are other distributions (e.g. with the same correlation) for which this limit is non-zero. By changing the signs of the random variables in this conditional probability, the tail areas in the other three quadrants can be investigated analogously.

As we consider each tail separately, shocks in X and Y are not averaged out, whereas this averaging may hamper regression analysis. Consider the following somewhat contrived example specification. Suppose

$$Y = X + (\vartheta - 2\xi) \tag{21}$$

and where the explanatory variable X is driven by two random shocks

$$X = \zeta + \xi. \tag{22}$$

The error term in equation (21) is also driven by two shocks ϑ and ξ. Shocks in ζ do affect both X and Y in the same positive way. A shock in ϑ does affect Y, but does not show up in X; a shock in ξ does affect both X and Y, but in opposite directions. A shock in ξ increases X, but in equation (21) this effect of X on Y is overwhelmed by the negative impact of ξ in the noise term of Y. In equation (19), this can occur if the monetary policy shocks, e.g., a higher growth rate of money supply, increase both Δx_{t-1} and the risk premium ρ_{t-1}, as these appear with opposite sign in the solution for the exchange rate returns.

For illustration purpose, suppose that all three shocks are independently Student-t distributed with the same degrees of freedom $\alpha > 2$. As a result $\sigma_\xi^2 = \sigma_\zeta^2 = \sigma_\vartheta^2$, while the covariances are zero. An OLS regression gives

$$\widehat{\beta} = \frac{Cov[\zeta - \xi + \vartheta; \zeta + \xi]}{Var[\zeta + \xi]} \simeq \frac{\sigma_\zeta^2 - \sigma_\xi^2}{\sigma_\xi^2 + \sigma_\zeta^2} \to 0$$

in large samples. Similarly, a quantile regression would also not elicit any dependence.

Alternatively, consider the limit conditional probability in equation (20). Below we demonstrate that this gives the following limit

$$\begin{aligned}\frac{\Pr\{X>q,Y>q\}}{\Pr\{X>q\}} &= \frac{\Pr\{\zeta+\xi>q,\zeta-\xi+\vartheta>q\}}{\Pr\{\zeta+\xi>q\}} \\ &\simeq \frac{\Pr\{\zeta>q,\zeta>q\}}{\Pr\{\zeta+\xi>q\}} \rightarrow \frac{1}{2} \end{aligned} \tag{23}$$

for large q (quantiles). Thus the tail analysis shows that Y does depend on X, while a regression analysis would suggest this is not the case. The regression analysis is hampered by endogeneity as the shocks $\vartheta - 2\xi$ in equation (21) are correlated with the explanatory variable X.

In this article, we use limit copula as an alternative to regression analysis to evaluate the exchange rate models and examine the association in the tails between exchange rate returns and lagged macroeconomic fundamentals. While regression analysis captures the average relationship between variables Y and X, here we assess the extreme linkages between the variables per quadrant. By considering each tail separately, shocks in X and Y are not averaged out as might occur in regression analysis.

2.3 Extreme Linkages

In this section, by using statistical extreme value analysis we explain in further detail the extreme linkages implied by the exchange rate model.

2.3.1 Convolution Theorem

Above we demonstrated that the distribution of individual macroeconomic variables can exhibit a power law, i.e.

$$\Pr\{X>q\} = 1 - F(q) \sim Aq^{-\alpha}, \text{ as } q \rightarrow \infty.^{9} \tag{24}$$

How does this translate to the composite fundamental x? According to Feller's Convolution Theorem (1971, VIII.8), if X_1 and X_2 are independent with common c.d.f. $F(q)$ from (24), then

$$\Pr\{X_1+X_2>q\} \sim 2Aq^{-\alpha}, \text{ as } q \rightarrow \infty. \tag{25}$$

Thus, to a first order at large thresholds q the probability of the sum equals the sum of the marginal probabilities. The main takeaway is that to a first order at large quantile levels q, only the univariate probability mass along the axes in the (X_1, X_2)-plane contributes to the probability of a large realization.

As is explained in Appendix B, this result also applies if the random variables are not independent but exhibit multivariate regular variation (which is the multivariate analogue of the power law assumption (24)). A further result is that if the tail indices α differ, the random variable with the thickest tail (smallest α) dominates the sum that defines the composite fundamental. In short, if individual macroeconomic variables exhibit power law behavior, so does the distribution of the sum of these random variables. The monetary model is linear in the fundamentals and hence the convolution theorem applies if (some of) the fundamental variables do have a distribution with a heavy tail. Note that the power law in exchange rate returns does not necessarily stem from the fundamentals, but may be due to the noise terms.

[9] The expression $A \sim B$ means that $\lim (A/B) = 1$.

To see the difference between the sum of heavy-tailed variables and the sum of thin-tailed variables, suppose that in the above example (21) and (22), the (ζ, ϑ, ξ) are independent Student-t distributed random variables with the same $\alpha > 2$ degrees of freedom (so that means and variances exist). Then for large q

$$\Pr\{|\zeta| > q\} = \Pr\{|\vartheta| > q\} = \Pr\{|\xi| > q\} \sim 2cq^{-\alpha},$$

where

$$c = \Gamma\left(\left(\alpha+1\right)/2\right)\alpha^{(\alpha-1)/2}/\Gamma\left(\alpha/2\right)\sqrt{\alpha\pi}.$$

The absolute value and symmetry of the tails explains the factor 2 in the expression, thus for the upper tail for example $\Pr\{\xi > q\} \sim cq^{-\alpha}$.

By Feller's convolution theorem (see Appendix B for details and a simplified proof)

$$\Pr\left\{\zeta + \xi > q\right\} \sim 2cq^{-\alpha} \tag{26}$$

and

$$\Pr\left\{\zeta - \xi + \vartheta > q\right\} \sim 3cq^{-\alpha}.$$

For the joint distribution $\Pr\left\{\zeta + \xi > q, \zeta - \xi + \vartheta > q\right\}$, notice that the shocks ξ appear with opposite signs. Therefore a large positive shock in ξ likely contributes to an extreme realization of the sum $\zeta + \xi$, but has the opposite effect on $\zeta - \xi + \vartheta$. A slight extension of the Feller theorem then shows that for large q

$$\Pr\left\{\zeta + \xi > q, \zeta - \xi + \vartheta > q\right\} \sim \Pr\left\{\zeta > q, \zeta > q\right\} \sim cq^{-\alpha}.$$

Combining numerator and denominator gives the result in equation (23).

Alternatively, if the (ζ, ϑ, ξ) are independent mean zero standard normal shocks, then the sum in equation (26) is also normally distributed. Laplace's classical expansion for the tail probabilities is the density to the (large) quantile, which immediately shows that the tail probabilities are of exponential nature. Briefly, if say both ζ and ξ are independent standard normally distributed, then for large q

$$\Pr\{\zeta > q\} = \Pr\{\xi > q\} \sim \frac{1}{q}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}q^2},$$

so that

$$\Pr\{\zeta + \xi > q\} \sim \frac{1}{q}\frac{1}{\sqrt{\pi}}e^{-\frac{1}{4}q^2}, \text{ as } q \to \infty.$$

The important difference between summing the Student-t random variables and the normal case is that in the former case the power α remains as it is, but in the normal case the power changes from $-1/2$ in the exponent to $-1/4$. In the latter case, the probability approaches zero at a slower rate. Thus, it follows that

$$\lim_{q\to\infty}\frac{\Pr\{\zeta > q\}}{\Pr\{\zeta + \xi > q\}} = \lim_{q\to\infty}\frac{1}{\sqrt{2}}e^{\left(-\frac{1}{2}+\frac{1}{4}\right)q^2} = 0.$$

This explains why the dependency between normally distributed random variables eventually vanishes. For more details, see e.g., De Vries (2005). In the above example, since the two random variables $\zeta + \xi$ and $\zeta - \xi + \vartheta$ are uncorrelated, the two normally distributed random variables are in this case also independent and hence

$$\begin{aligned}
\Pr\left\{\zeta + \xi > q, \zeta - \xi + \vartheta > q\right\} &= \Pr\left\{\zeta + \xi > q\right\}\Pr\left\{\zeta - \xi + \vartheta > q\right\} \\
&\sim \frac{1}{q}\frac{1}{\sqrt{\pi}}e^{-\frac{1}{4}q^2} * \frac{\sqrt{3}}{q}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{6}q^2} \\
&= \frac{\sqrt{3/2}}{q^2}\frac{1}{\pi}e^{-\frac{5}{12}q^2}
\end{aligned}$$

which tends to zero faster than $\Pr\{\zeta + \xi > q\}$. So that there is no asymptotic dependence.

Thus we can expect heavy tails in the exchange rate return distribution if the fundamentals have distributions that are heavy tailed. Moreover, this induces asymptotic dependence between the right-hand-side and left-hand-side variables of equation (19), i.e. conditional on large macroeconomic fundamental shocks, the probability of extreme currency swings is positive. In general, it is not the case that if the marginal distributions have heavy tails, the random variables are necessarily asymptotically dependent. For example, Student-t distributed random variables combined with a Gaussian copula, are correlated but asymptotically independent. The linearity of the model (19) combined with the marginal heavy tail feature, however, does induce the asymptotic dependency between Δs on the one hand and the Δx_{t-1} components and their contemporaneous shocks on the other hand. The dependency is preserved in the tail area, and even if the correlation coefficient is equal to 0, there can still be asymptotic dependence.

2.3.2 Asymptotic Dependence and Independence

The way in which the asymptotic dependency between two random variables Y and X can be expressed is through the conditional tail probability

$$\lim_{q\to\infty} \frac{\Pr\{X > q, Y > q\}}{\Pr\{X > q\}} > 0. \tag{27}$$

If instead X and Y are dependent, but the limit is zero, one speaks of asymptotic independence. This occurs for example if, like in the example above, the random variables are bivariate standard normally distributed and where the correlation coefficient is not equal to 0, -1 or 1. The measure (27) was first proposed by Huang (1992), and is extensively discussed in McNeil, Frey and Embrechts (2005). It is used in Poon, Rockinger and Tawn (2004), Hartmann, Straetmans and de Vries (2010) and Cumperayot and Kouwenberg (2013). At a finite quantile level q the measure gives the probability of a joint excess, given that one of the two random variables exceeds q. One might ask what the relevance is of evaluating this probability in the limit. One contribution of statistical extreme value theory is that it shows that the limit conditional probability is a good approximation for values at finite levels of q in the range of the tail area of the joint distribution.

In equation (27), the dependency is measured along the diagonal, but one can measure along different rays by scaling the quantiles q for X and Y differently. A popular alternative in applied work, see e.g. Poon et al. (2004), is to use the same probability level p, instead of using the same quantile level q. This works as follows. Define the quantiles q_x and q_y:

$$\Pr\{X > q_x\} = \Pr\{Y > q_y\} = p.$$

The alternative measure evaluates

$$\lim_{p\downarrow 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{\Pr\{X > q_x(p)\}} = \lim_{p\downarrow 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{p}. \tag{28}$$

Note that $q_x(p)$ and $q_y(p)$ are generally different. If the scale of the random variables differs considerably it makes sense to use the measure (28). We therefore follow the applied literature and measure the extreme linkage conditioning on the same probability level. Results using the same quantiles are available upon request. Similar measures exist for the other quadrants by simply switching the signs of X and/or Y.

If one finds no support for asymptotic dependence this can be due to one of the following two explanations. One possibility is that the fundamentals based exchange rate model does not apply, so that the noise is exogenous and is unrelated to the macroeconomic fundamentals. Alternatively, even if two random variables are (imperfectly) correlated, but follow e.g. a multivariate normal distribution, then all dependency vanishes asymptotically. Thus if we reject asymptotic dependence, there are two possible explanations. If we find that asymptotic dependence is not rejected, this at least suggests a strong linkage between the fundamentals and the exchange rate returns via the (larger) shocks that drive both variables.

2.3.3 Asymptotic Dependence and Exchange Rate Model

In this subsection, we give an example to demonstrate the asymptotic linkage between the exchange rate returns and the macroeconomic fundamentals as implied by theoretical exchange rate model. Recall the fundamentals x_t from equation (14)

$$x_t = m_t + z_t - \gamma y_t - \nu_t.$$

We split the fundamentals into two groups, in line with the macroeconomic model that predicts a different distribution for real income innovations and the monetary related innovations. In line with the stochastic process (16) let $\phi < 1$, but consider the two separate stochastic processes

$$\Delta\left(m_t + z_t - \nu_t\right) = \phi\Delta\left(m_{t-1} + z_{t-1} - \nu_{t-1}\right) + \vartheta_t,$$

and

$$\Delta y_t = \phi\Delta y_{t-1} + \xi_t.$$

Thus the innovations ε in equation (16) are split into ϑ and ξ and are attributed to different parts of the fundamentals vector. Otherwise the process for the fundamentals is as in equation (16). The deviations from UIP follow the process in equation (17), i.e.

$$\rho_t = a\rho_{t-1} + \mu_t;\ a < 1.$$

Given a relationship between risk premiums ρ and traditional exchange rate fundamentals Δx, as in e.g. Sarno et al. (2012), we assume that the innovations μ are a composite of the innovations ϑ, ξ and ζ, i.e.

$$\mu_t \equiv \tau\vartheta_t + \kappa\xi_t + \zeta_t,$$

where ϑ, ξ and ζ are i.i.d., independent from each other, and have zero mean. The first two shocks are related to the fundamentals, while the third innovation is unrelated to the fundamentals of the monetary model.

To derive the extreme linkage in the first quadrant, assume that all the innovations follow Student-t distributions with the same degrees of freedom $\alpha > 2$ and unit scale. From equation (19), the joint probability of the exchange rate returns and lagged fundamentals conditional on time $t-2$ information becomes

$$\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q\right|\Delta x_{t-2}, \rho_{t-2}\big\} =$$
$$\Pr\left\{(1-b)\frac{\phi}{1-b\phi}\Delta x_{t-1} - \frac{(1-a)\,b}{1-ab}\rho_{t-1} + \frac{b}{1-ab}\mu_t + \frac{1}{1-b\phi}\left(\vartheta_t - \gamma\xi_t\right), \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\}.$$

Engel and West (2005) discuss the case in which the discount factor is close to unity. For simplicity, take this to the extreme and suppose that $b = 1$ in fact, then

$$\begin{aligned}
&\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q\right|\Delta x_{t-2}, \rho_{t-2}\big\} \\
&= \Pr\left\{-\rho_{t-1} + \frac{1}{1-a}\mu_t + \frac{\vartheta_t - \gamma\xi_t}{1-\phi} > q, \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\} \\
&= \Pr\left\{-\rho_{t-1} > q, \Delta x_{t-1} > q\right|\Delta x_{t-2}, \rho_{t-2}\big\}.
\end{aligned}$$

Iterating one period back gives

$$\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\}$$

$$\begin{aligned} &= \Pr\left\{-\left(a\rho_{t-2} + \tau\vartheta_{t-1} + \kappa\xi_{t-1} + \zeta_{t-1}\right) > q, \phi\Delta x_{t-2} + \vartheta_{t-1} - \gamma\xi_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\} \\ &= \Pr\left\{-\tau\vartheta_{t-1} - \kappa\xi_{t-1} + \zeta_{t-1} > q, \vartheta_{t-1} - \gamma\xi_{t-1} > q\right\} \\ &\sim \Pr\left\{-\tau\vartheta_{t-1} - \kappa\xi_{t-1} > q, \vartheta_{t-1} - \gamma\xi_{t-1} > q\right\}. \end{aligned}$$

Conditional on the information at time $t-2$, ρ_{t-2} and Δx_{t-2} do not contribute to the shocks at $t-1$. Moreover, since the risk premium shock ζ_{t-1} is unrelated to the fundamentals, it does not contribute to Δx_{t-1} at large levels of q (under the assumption that all innovations are Student-t distributed).

First, suppose that both τ and κ are negative, i.e. a case in which positive shocks on the fundamentals reduce the UIP deviation ρ. In this case the relation further simplifies

$$\begin{aligned} &\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\} \\ \sim\ &\Pr\left\{-\tau\vartheta_{t-1} - \kappa\xi_{t-1} > q, \vartheta_{t-1} - \gamma\xi_{t-1} > q\right\} \\ \sim\ &\Pr\left\{-\tau\vartheta_{t-1} > q, \vartheta_{t-1} > q\right\}. \end{aligned}$$

Since both τ and κ are assumed to be negative and γ is positive, the shocks to income ξ run in opposite directions for Δs and Δx_{-1}. Only monetary shocks ϑ contribute to large positive levels of Δs and Δx_{-1}. The joint probability is asymptotic to

$$\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\} \sim \begin{cases} cq^{-\alpha} \ if \ -\tau > 1 \\ c\left(-\tau\right)^{\alpha} q^{-\alpha} \ if \ -\tau < 1 \end{cases},$$

where c is a scale factor determined by the specifics of the Student-t distribution. So that even if $b = 1$ the probability of a joint excess of large currency swings and fundamentals shocks is not necessarily equal to zero. The tail analysis indicates the potential association between the exchange rate returns and macroeconomic fundamentals, while in this case a regression analysis might be impaired by the high discount factor b.

Next consider the case in which τ and κ have opposite signs. Suppose that the risk premium on US exchange rates is countercyclical to the US economy, to the effect that ρ reacts negatively to monetary impulses and positively to relative output growth. This is the case for which Sarno et al. (2012) find considerable evidence. In this case, τ is negative and κ is positive. Then the asymptotic dependency is somewhat more complex

$$\begin{aligned} &\Pr\left\{\Delta s_t > q, \Delta x_{t-1} > q \middle| \Delta x_{t-2}, \rho_{t-2}\right\} \\ \sim\ &\Pr\left\{-\tau\vartheta_{t-1} - \kappa\xi_{t-1} > q, \vartheta_{t-1} - \gamma\xi_{t-1} > q\right\} \\ \sim\ &\left\{\min\left[1, (-\tau)^{\alpha}\right] + \min\left[\gamma^{\alpha}, \kappa^{\alpha}\right]\right\} cq^{-\alpha}. \end{aligned}$$

The intuition for this result is that the asymptotic dependency is determined by the amount of univariate probability mass located along the ϑ_{t-1} and ξ_{t-1} axes and satisfies the two inequalities $-\tau\vartheta_{t-1} - \kappa\xi_{t-1} > q$ and $\vartheta_{t-1} - \gamma\xi_{t-1} > q$.

In summary, we theoretically demonstrated that with multiplicative parameter uncertainty, the monetary macroeconomic fundamentals have distributions with power type tails. Conventional exchange rate models then imply that this heavy tail feature is transmitted to the distribution of the exchange rate returns, through the linearity of the models.

3 Estimation

In this section, we explain how we estimate and test for tail fatness and asymptotic dependence. Specifically, we apply the DEdH tail index estimator and the measure of asymptotic dependence from de Haan and Ferreira (2007).

3.1 Tail Index Estimator

From Dekkers, Einmahl and De Haan (1989), let $X_{(i)}$ be the descending order $X_{(1)} \geq X_{(2)} \geq ... \geq X_{(n)}$ from the sample of size n. Consider the upper tail, define the first two conditional log-moments

$$H = \frac{1}{M} \sum_{i}^{M} \log \frac{X_{(i)}}{X_{(m)}}$$

and

$$K = \frac{1}{M} \sum_{i}^{M} (\log \frac{X_{(i)}}{X_{(m)}})^2,$$

where $X_{(m)}$ is a suitable threshold and there are M observations above the threshold. Note that H is the familiar Hill (1975) tail index estimator, which is predicated on heavy tails.

The Dekkers, Einmahl and De Haan (1989) or DEdH estimator for the inverse of the tail index ($\gamma = 1/\alpha$) reads

$$\widehat{\gamma} = 1 + H + \frac{1}{2} \frac{\frac{K}{H}}{H - \frac{K}{H}}$$

and

$$\sqrt{M}\,(\widehat{\gamma} - \gamma)$$

is asymptotically normally distributed with variance $1 + \gamma^2$ (as long as $\gamma \geq 0$, i.e. as long as the support of the distribution is unbounded).

The Hill estimator has been shown to be asymptotically unbiased and more efficient than alternative estimators, including the DEdH estimator (see, e.g., Koedijk et al., 1992). But it only applies if the data are heavy tailed, i.e. if $\gamma > 0$. The advantage of the DEdH estimator is that it applies to all types of tails. Specifically, in the case the distribution exhibits heavy tails, $\gamma = 1/\alpha > 0$. In the case the tails are exponentially thin as in the case of e.g. the normal distribution, $\gamma = 0$; and in the case that the support is bounded, $\gamma < 0$. The disadvantage is that the variance of the DEdH estimator exceeds the variance of the Hill estimator in the case $\gamma = 1/\alpha > 0$. Therefore, while the DEdH estimator allows us to test for tail fatness, the estimator is more conservative in rejecting the null hypothesis of a thin tail.

3.2 Asymptotic Dependence Measure

This subsection provides the non-parametric estimation methodology of the extreme linkage measure. To estimate the dependence measure (28), we use a simple count estimator to evaluate

$$\lim_{p \downarrow 0} \frac{\Pr\{X > q_x(p), Y > q_y(p)\}}{p}. \tag{29}$$

Only a subset k with the more extreme observations from the tail area are used for estimation. Specifically, let n be the sample size and k be a sequence of numbers such that $k(n) \to \infty$ as $n \to \infty$, but $k(n)/n \to 0$. The probability p is proxied by $k/(n+1)$. Let $X_{(i)}$ and $Y_{(i)}$ denote the descending order statistics of X_i and Y_i. The corresponding empirical distribution functions are respectively $F_n(x)$ and $G_n(y)$. The empirical counterpart for the denominator thus reads

$$p \simeq 1 - F_n(X_{(k)}) = 1 - G_n(Y_{(k)}) = \frac{k}{n+1}.$$

For the numerator we count the number pairs (X_i, Y_i) for which both $X_i \geq X_{(k)}$ and $Y_i \geq Y_{(k)}$ and divide by the sample size. This gives the following count estimator

$$\widehat{S\left(k\right)} = \frac{\frac{1}{n+1}\sum_{i=1}^{n} 1_{\{X_i \geq X_{(k)},\, Y_i \geq Y_{(k)}\}}}{k/\left(n+1\right)} = \frac{1}{k}\sum_{i=1}^{n} 1_{\{X_i \geq X_{(k)},\, Y_i \geq Y_{(k)}\}}, \tag{30}$$

where $1_{\{\}}$ is the indicator function.

The asymptotic confidence band follows from de Haan and Ferreira (2007, ch.7). Specifically, define

$$\widehat{L\left(k\right)} = \frac{1}{k}\sum_{i=1}^{n} 1_{\{X_i \geq X_{(k)},\text{ or } Y_i \geq Y_{(k)}\}}.$$

Note that

$$\widehat{S\left(k\right)} = 2 - \widehat{L\left(k\right)}.$$

So that the asymptotic variance of $\widehat{S}$ equals the asymptotic variance of $\widehat{L}$. The latter variance is estimated by means of

$$\widehat{\sigma}^2_{\widehat{L}} = \widehat{L\left(k\right)} + \left(L_1^2 - 2L_1\right) + \left(L_2^2 - 2L_2\right) + 4L_1L_2\left(2 - \widehat{L}\right),$$

where

$$L_1 = \frac{1}{k^{3/4}}\sum_{i=1}^{n} 1_{\{X_i \geq X_{\left(k+k^{3/4}\right)},\text{ or } Y_i \geq Y_{(k)}\}} - k^{1/4}\widehat{L\left(k\right)}$$

and

$$L_2 = \frac{1}{k^{3/4}}\sum_{i=1}^{n} 1_{\{X_i \geq X_{(k)},\text{ or } Y_i \geq Y_{\left(k+k^{3/4}\right)}\}} - k^{1/4}\widehat{L\left(k\right)}.$$

Since $k + k^{3/4}$ can be non-integer, this is rounded to the nearest integer. De Haan and Ferreira (2007, ch.7) prove that

$$\sqrt{k}\left(\widehat{L\left(k\right)} - L\right)$$

is asymptotically normally distributed with mean zero and variance $\widehat{\sigma}^2_{\widehat{L}}$. From this, confidence bands can be easily constructed.

In Figure 1, we illustrate the plots of $\widehat{S(k)}$ and its asymptotic confidence band for different thresholds k using simulated data with correlation of 0.7.[10] The plot in the left panel shows the estimates for Student-t random variables with 3 degrees of freedom, while the right panel displays normally distributed random variables. From the figure, we observe that the Student-t based plot immediately jumps up, but the normal based plot only gradually rises from a lower level. The difference between these two cases becomes more obvious once we zoom in, see Figure 2. For the Student-t random variables, the plot of $\widehat{S(k)}$ immediately jumps to a stable plateau, where it lingers around 0.5. This indicates the case of asymptotic dependence. For asymptotic independence, the plot often stays in the neighborhood of 0 before slowly rising towards 1 at the end.

[10] To be specific, the two series of simulated uniformly distributed random variables are transformed to Student-t random variables with 3 degrees of freedom and normally distributed random variables, both with correlation coefficient of 0.7.

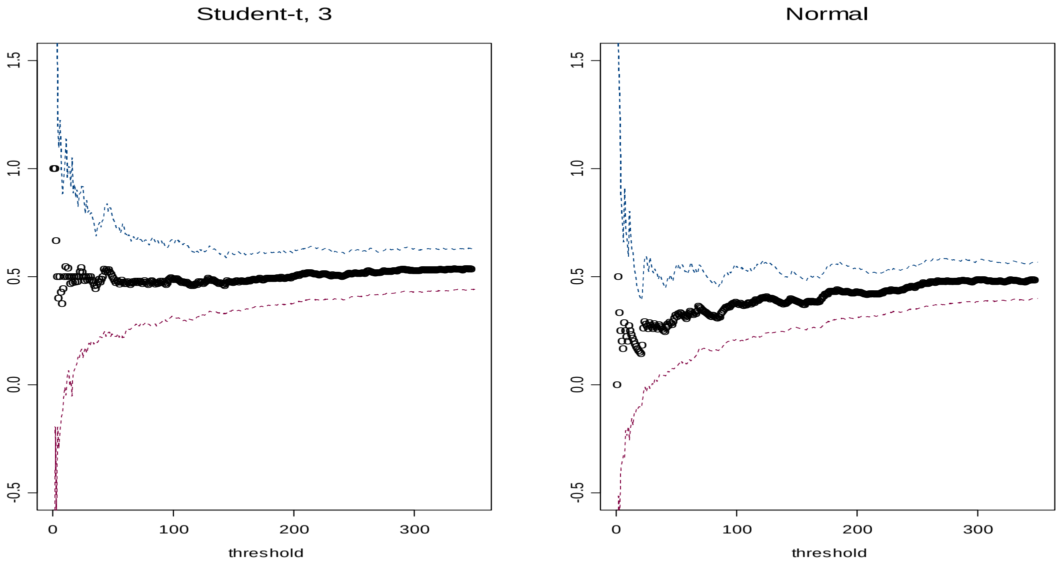

Figure 1: Simulated Student-t and Normal data with correlation 0.71

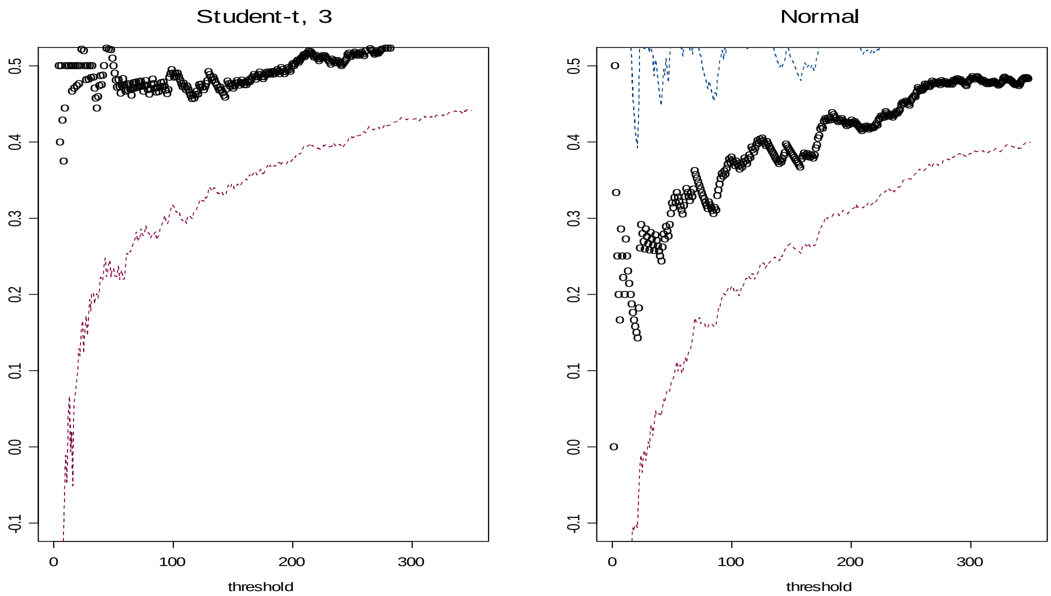

Figure 2: Zoom of Figure 1

4 Empirical Results

Our empirical aim is twofold. First, we examine whether or not some of the macroeconomic fundamental shocks are heavy tail distributed. Second, if this is the case, we investigate the asymptotic dependency between the variables on both sides of the exchange rate model. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries in three groups.[11] Observations used to estimate and test for the fat tails and the extreme linkage between exchange rate returns and economic fundamentals then range from 3843 to 6096. The panel analysis of the joint tail events is partly justified by the similarity of tail indices across countries.[12]

Furthermore, we study the extreme linkage between exchange rate returns and lagged *individual* fundamentals. This is similar to the regression analysis in Engel and West (2005), with an attempt to conduct a relatively unstructured investigation of the link between exchange rates and macroeconomic fundamentals. A problem with using the composite fundamental Δx_{t-1} in practice, is that the composite relies on knowing coefficients like γ on the individual fundamentals. Measuring these coefficients is, however, difficult. Moreover, different theories tend to suggest different sets of fundamentals. In this article, we identify the relevance of each fundamental separately. We examine whether large movements of macroeconomic fundamentals are associated with large currency swings, and which fundamentals matter most for the extreme movements of exchange rates.

The extreme value theory discussed earlier offers an advantage over a regression based analysis when part of the model under the null hypothesis is not well specified. It helps circumvent the need to first measure the contribution of the specific fundamental to the composite. Nevertheless, extreme value analysis itself can be rather sensitive to the number of observations included in the tail area. Too few observations can enlarge the variance of the estimate, while too many observations reduce the variance at the expense of biasedness due to including observations from the central range. Below we report the estimates using a typical tail size of 2.5% of the overall sample. The main conclusions of our study are still valid when using 5% or 1% of the overall sample as the tail data. The results are available upon request.

4.1 Tail Indices

In Table 1, we report the estimates of the inverse tail index , i.e. $\gamma = 1/\alpha$, and asymptotic 95% confidence intervals (in parentheses) using the DEdH estimator. The higher the γ implies the fatter the tail, while the integer value of the tail index α indicates the number of bounded moments.[13] The DEdH tail estimator has the advantage that it applies to all types of tails. Its disadvantage in comparison to estimators that directly presume that the data are heavy tailed, is that it can be quite variable depending on the number of observations that are used. The fact that the estimator is asymptotically normally distributed allows one to test for the type of tail. If $\gamma = 0$, asymptotic dependency is ruled out.

Table 1 gives the DEdH estimates using the 2.5% tail observations of the overall sample. The right tail indicates large depreciations of the domestic currency and dramatic increases in the domestic fundamentals relative to their foreign country, while the left tail shows the opposite pattern. In the case of Asia and Latin America, the domestic variables are relative to the US variables, and the US dollar (USD) is a base currency. For the European countries, apart from

[11] The data set and descriptive statistics are shown in Appendix A.
[12] To save space, estimated tail indices for individual countries are available upon request.
[13] A more detialed explanation of the tail shape parameter α is provided in equation (31) in Appendix B.

the US we also consider the domestic variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The Deutsche mark (DM), and thereafter the euro (EUR) are then a base currency. The estimator is used as a pre-test for the tail type. If the confidence band ranges from negative to positive, the possibility of an exponentially thin tail or endpoint distribution cannot be excluded.

For Asian and Latin American countries, all exchange rate returns and changes in macroeconomic fundamentals exhibit heavy tails. In almost all cases, the estimated γ is higher than 0.25 which indicates that the fourth moment does not exist. Half of the Asian and Latin American variables have γ larger than 0.5, i.e. a tail index α below 2. It implies that in these cases the variance of the variables is not bounded. The variables are very heavy tailed, compared to the thin-tailed normal distribution which has all moments bounded. Apparently, for the Asian and Latin American countries we also find evidence that real income exhibits heavy tails. This can, however, not be due to the mechanism that explains the heavy tail feature of the monetary variables that arises from the Brainard uncertainty.

For the European countries, there are only a number of cases in which the variables significantly exhibit heavy tails. When considered relative to the US, only the negative growth rates of money supply and price level, and both positive and negative changes in the interest rates have heavy tails. When the German Mark or the euro is used as the base currency, the heavy tail cases are the depreciation of the European currencies, the positive growth rate of money supply and both tails of the interest rate changes.

In sum, Table 1 shows that not only exchange rate returns have heavy tails, but also macroeconomic fundamentals. Furthermore, both nominal as well as real variables appear to be heavy tailed. The tail fatness, however, varies from one region to another. We have discussed in Section 2 that given the linearity of the exchange rate model, the presence of large shocks (or heavy tails) on both sides of equation (19) implies a specific kind of dependency, i.e. asymptotic dependence. Next, we empirically estimate and test for the asymptotic dependence.

4.2 Extreme Linkages

To test the hypothesis that large swings in exchange rates are linked to large changes in the heavy-tailed macroeconomic fundamentals, we apply the linkage estimator (30) which conditions on the same probability level, see equation (28).[14] Using 2.5% of the data, Table 2 and 3 give the estimates of the linkage measure and asymptotic 95% confidence intervals (in parentheses) for large depreciations and appreciations of the domestic currency, respectively. The depreciation side is on the right tail of the exchange rate returns' distribution, and opposite for the appreciation. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables.

For instance, $\Delta s, \Delta m_{-1}, +$ represents the positive relation between exchange rate returns and lagged relative money supply growth. In Table 2, the results test the linkage between the right tails of the distributions of exchange rate returns and money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to abroad. The linkage between their left tails, showing large appreciations and declines in relative money supply, is shown in Table 3. For the interest rate Δi_{-1}, theory is ambiguous regarding the relation between the exchange rate and the interest rate differential. We therefore examine both positive $+$ and negative $-$ relations.

For the European fundamentals and exchange rate returns, both tables show that the limit conditional probability is close to zero and insignificant in all cases regardless of the base currency.

[14] The results conditioning on the same quantile level are available upon request.

Table 1: DEdH Estimates of Inverse Tail Index

	Base Currency	Europe DM/EUR	Europe USD	Asia USD	Latin America USD
Δs	left	0.0593	-0.0311	0.2975	1.2338
		(-0.1380, 0.2567)	(-0.2173, 0.1550)	(0.1321, 0.4628)	(0.9554, 1.5123)
	right	0.3122	-0.2747	0.4789	0.8379
		(0.1058, 0.5185)	(-0.4676, -0.0817)	(0.3032, 0.6546)	(0.6092, 1.0666)
Δm	left	0.1396	0.2110	0.5889	0.5787
		(-0.0624, 0.3416)	(0.0145, 0.4074)	(0.3987, 0.7791)	(0.3745, 0.7829)
	right	0.3973	0.1473	0.3945	0.1890
		(0.1820, 0.6125)	(-0.0470, 0.3416)	(0.2183, 0.5707)	(0.0091, 0.3689)
Δy	left	-0.0648	-0.5574	0.5883	0.4692
		(-0.2622, 0.1326)	(-0.7713, -0.3434)	(0.3896, 0.7869)	(0.2493, 0.6890)
	right	-0.1030	-1.0792	0.5619	0.3489
		(-0.3010, 0.0950)	(-1.3542, -0.8043)	(0.3654, 0.7583)	(0.1382, 0.5597)
Δp	left	0.1478	0.3698	0.1693	0.3588
		(-0.0513, 0.3469)	(0.1714, 0.5681)	(0.0054, 0.3333)	(0.1718, 0.5458)
	right	0.0558	0.1701	0.3082	0.2807
		(-0.1415, 0.2531)	(-0.0187, 0.3588)	(0.1391, 0.4774)	(0.0979, 0.4636)
Δi	left	0.5494	0.4704	0.9421	1.7135
		(0.3223, 0.7765)	(0.2629, 0.6779)	(0.7185, 1.1657)	(1.3571, 2.0700)
	right	0.4418	0.2928	0.9747	1.7089
		(0.2242, 0.6594)	(0.0972, 0.4884)	(0.7474, 1.2019)	(1.3532, 2.0647)

Table 1 shows the DEdH estimates of the inverse tail index and asymptotic 95% confidence intervals (in parentheses) using the 2.5% tail observations of the overall sample. The variables are the exchange rate returns s, the rate of change in relative money supply m, the rate of change in relative real income y, the rate of change in relative price p and change in the interest rate differential i. The right tail indicates large depreciations of the domestic currency and dramatic increases in the domestic fundamentals relative to their foreign country, while the left tail shows the opposite pattern. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are then a base currency.

Table 2: Estimates of Extreme Linkage

	Depreciation			
	Europe	Europe	Asia	Latin America
Base Currency	DM/EUR	USD	USD	USD
$\Delta s, \Delta m_{-1}, +$	0.0104	0.0194	0.2676	0.2276
	(-0.0250, 0.0458)	(-0.0263, 0.0652)	(0.1437, 0.3915)	(0.1121, 0.3432)
$\Delta s, \Delta y_{-1}, -$	0.0510	0.0364	0.0229	0.0417
	(-0.0217, 0.1237)	(-0.0254, 0.0982)	(-0.0215, 0.0673)	(-0.0287, 0.1120)
$\Delta s, \Delta p_{-1}, +$	0.0404	0.0636	0.2466	0.2358
	(-0.0269, 0.1077)	(-0.0181, 0.1454)	(0.1337, 0.3594)	(0.1213, 0.3502)
$\Delta s, \Delta i_{-1}, +$	0.0928	0.0833	0.1862	0.2373
	(-0.0126, 0.1982)	(-0.0095, 0.1761)	(0.0675, 0.3049)	(0.1156, 0.3590)
$\Delta s, \Delta i_{-1}, -$	0.0309	0.0648	0.1034	0.1017
	(-0.0280, 0.0898)	(-0.0184, 0.1481)	(0.0174, 0.1895)	(0.0049, 0.1985)

Table 2 shows the estimates of the extreme linkage measure and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data for large depreciations of the domestic currency relative to the base currency. The first column shows the pair of variables under investigation. The variables are the exchange rate returns s, the rate of change in relative money supply m, the rate of change in relative real income y, the rate of change in relative price p and change in the interest rate differential i. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns' distribution. Hence, s,m,+ represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to abroad. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

This may be due to the fact that we do not find much evidence of heavy tails for the European variables. For Asian and Latin American currencies, we however find strong evidence for asymptotic dependence between depreciations of the domestic currency and increases in money supply Δm_{-1}, prices Δp_{-1} and interest rates Δi_{-1}. Yet, for depreciations of the domestic currency and declines in real output Δy_{-1}, the link is not significant in any of the regions; the estimates are close to zero and the null hypothesis of asymptotic independence cannot be rejected. Large depreciations of the domestic currency are mainly a monetary phenomenon, and strong links are found for Asian and Latin American currencies. Interestingly, even though Table 1 suggests income may be heavy tailed for Latin American and Asian countries, we do not find evidence for asymptotic dependency between real income changes and exchange rate returns. The absence of tail dependence for real income is in line with the model prediction; the heavy tail feature is not.

Estimates for the appreciation side are given in Table 3. The limit conditional probability is close to zero for all cases, even for the monetary variables, indicating asymptotic independence. Extreme contractionary monetary policy or high deflation do not significantly induce large appreciations of the domestic currency. Strong evidence of asymptotic dependence implied by exchange rate theories is only found on the depreciation side of the Asian and Latin American currencies and only for the monetary variables. Large increases in money supply, prices and interest rates are followed by dramatic depreciations of the domestic currency, with probabilities of more than 20% approximately. The evidence also shows that responses of the exchange rate returns to macroeconomic fundamental shocks are asymmetric.

Table 3: Estimates of Extreme Linkage

	Appreciation			
	Europe	Europe	Asia	Latin America
Base Currency	DM/EUR	USD	USD	USD
$\Delta s, \Delta m_{-1}, +$	0.0104	0.0485	0.0423	0.0407
	(-0.0245, 0.0453)	(-0.0214, 0.1185)	(-0.0153, 0.0998)	(-0.0185, 0.0998)
$\Delta s, \Delta y_{-1}, -$	0.0102	0.0273	0.0153	0.0104
	(-0.0243, 0.0447)	(-0.0255, 0.0800)	(-0.0213, 0.0518)	(-0.0256, 0.0465)
$\Delta s, \Delta p_{-1}, +$	0.0202	0.0364	0.0342	0.0732
	(-0.0270, 0.0674)	(-0.0239, 0.0967)	(-0.0167, 0.0852)	(-0.0060, 0.1523)
$\Delta s, \Delta i_{-1}, +$	0.0309	0.0463	0.0276	0.0763
	(-0.0289, 0.0908)	(-0.0204, 0.1130)	(-0.0183, 0.0734)	(-0.0092, 0.1617)
$\Delta s, \Delta i_{-1}, -$	0.0515	0.0093	0.0276	0.0593
	(-0.0255, 0.1286)	(-0.0230, 0.0416)	(-0.0186, 0.0737)	(-0.0147, 0.1334)

Table 3 shows the estimates of the extreme linkage measure and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data for large appreciations of the domestic currency relative to the base currency. The first column shows the pair of variables under investigation. The varibles are the exchange rate returns s, the rate of change in relative money supply m, the rate of change in relative real income y, the rate of change in relative price p and change in the interest rate differential i. Positive and negative signs indicate positive and negative relations between the two variables. The appreciation side is on the left tail of the exchange rate returns' distribution. Hence, s,m,1,+ represents the linkage between the left tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large appreciations of the domestic currency and decreases in domestic money supply relative to abroad. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

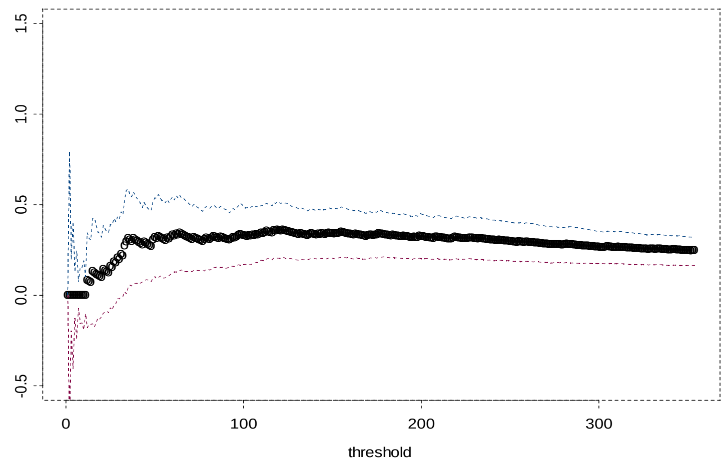

Figure 3: Asymptotic Dependency Between Asian Currency Depreciation and Relative Money Supply

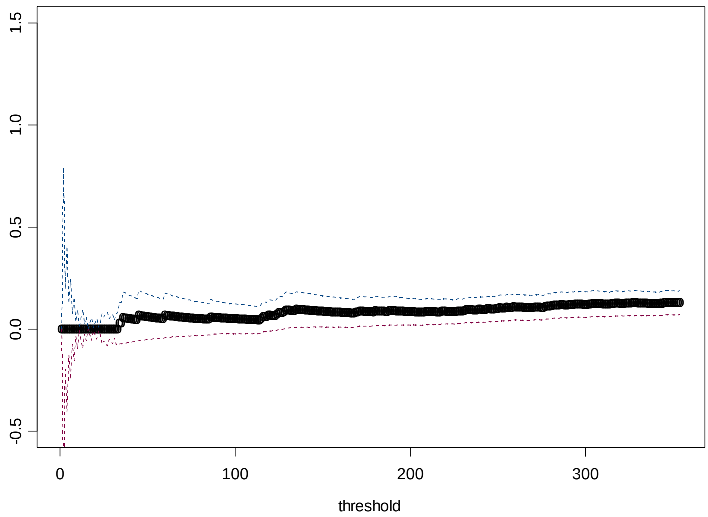

Figure 4: Asymptotic Independency Between Asian Currency Appreciation and Relative Money Supply

We end this subsection with two plots of the linkage estimator (30) for the relative money supply growth and exchange rate returns for the group of Asian countries. The plots in Figure 3 and Figure 4 illustrate the difference between asymptotic dependence and asymptotic independence, respectively. In Figure 3 we plot the case of Asian currency depreciations against the positive growth of relative money supply. Figure 4 shows the case of Asian currency appreciations against the negative growth of relative money supply. There is a marked difference between the two plots. For the depreciation side, the conditional probability almost immediately jumps upwards, as in the left panels of Figure 1 and 2. The graph for the appreciation has more resemblance with the right-hand-side panels of Figure 1 and 2. This suggests that conservative monetary policy and exchange rate returns are asymptotically independent, while highly inflationary money growth and exchange rates are asymptotically dependent.

In summary, we detect significant connections between large swings in currencies and macroeconomic fundamentals. The extreme linkages occur exclusively for Asian and Latin American currencies, not for Europe. Furthermore, we uncover asymmetric responses of the exchange rate returns to large changes in macroeconomic fundamentals, as the asymptotic dependence is only significant for the depreciation of the domestic currency. The monetary variable displays asymptotic dependence with the exchange rate depreciation. Therefore, the results lend further support to traditional exchange rate models in the vein of research initiated by Engel and West (2005) and Engel, Mark and West (2007).

4.3 Robustness Checks

We examine the robustness of the asymptotic links between exchange rate returns and macroeconomic fundamentals, considering several dimensions.[15] To summarize, we first lagged the macroeconomic fundamentals up to 12 lags and find that the estimates of the linkage measure do not change substantially, see Table A2. This is indicative for the persistence of tail dependence. Second, in Table A3 we show the estimates of the tail linkage between the *time t* macroeconomic fundamentals and *lagged* exchange rate returns. It follows from Sarno and Schmeling (2014), who argue that if exchange rates are driven by expected future fundamentals, then exchange rates contain information regarding future fundamentals. We find that those macroeconomic fundamentals that are asymptotically dependent with exchange rate returns in the previous subsection, are also significantly associated with the lagged exchange rate return. This confirms the extreme linkages between exchange rate returns and macroeconomic fundamentals, as suggested by the present-value model of exchange rates.

Third, we investigate whether the asymptotic links between exchange rate returns and macroeconomic fundamentals disappear when using a lower data frequency. When using quarterly data, as shown in Table A4 our estimation results remain the same but with higher estimated conditional probabilities. Then, in Table A5 we show that the associations between large swings in currency prices and lagged macroeconomic fundamental shocks, captured by the linkage measure (29), are extreme events in the economic sense. To do so, we use the exchange rate regime classification advanced by Ilzetzki, Reinhart and Rogoff (2008).[16] There are five coarse regimes; namely the pegged, limited flexibility, managed floating, freely floating and freely falling regimes. After excluding the freely falling regime, the asymptotic dependence between currency depreciations and increases in money supply, price and interest rate, though significant, declines from 20-30% to a single digit number.[17] The results thus indicate that the asymptotic dependence between exchange rate returns and monetary variables is strongly influenced by crisis episodes.

Last, we graphically illustrate the differences between pairs of the variables which are asymptotically dependent and independent, by plotting the probability of a large currency depreciation given a large macroeconomic shock. Figure A1 shows on the Y-axis the conditional probability $\Pr\{\Delta s > q_s(p) \,|\Delta x_{-1} > q_x(p)\}$ of a large currency depreciation given a large shock in the macroeconomic fundamental Δx_{-1}. The X-axis indicates $1-p$, with p the percentile of the fundamental variable Δx_{-1}. That is when the shock becomes large $q_x(p) \to \infty$, on the X-axis $1-p \to 1$. In Figure A1, the contrast between the cases of asymptotic dependence and independence, reported in Table 2 and 3, are clear. When the variables are asymptotically independent, e.g. the cases between Δs and Δy_{-1}, the conditional probability moves along the diagonal line and eventually approaches null as $q_y(p) \to \infty$. Yet, for the cases of asymptotically dependence, e.g. between Δs and Δm_{-1} of Asia and Latin America, the probability of joint extremes lingers at a positive number for large fundamental shocks. Thus, in those cases the conditional probability is non-zero even in the limit.

[15] The results are in Appendix C.

[16] We use the monthly coarse classification of exchange rate regimes from Ilzetzki, Reinhart and Rogoff (2008). For details on the de facto exchange rate regime classification, the reader is referred to Reinhart and Rogoff (2004) and Ilzetzki, Reinhart and Rogoff (2008).

[17] Note that observations are classified as freely falling when one of the following conditions applies. First, the annual inflation rate is above 40%, but excluding months during which the exchange rate still follows an official pre-announced arrangement (crawl or band). Second, the six months immediately follow a currency crisis, but only for the cases where the crisis marks a transition from a fixed (or quasi-fixed) regime to a managed or freely floating regime.

5 Conclusion

In standard theoretical exchange rate models, shocks to the macroeconomic fundamentals, like money supply, real income, price and interest rate, drive both the explanatory variables and the dependent variable. Exchange rate returns at times display large movements. An interesting question is whether there exists a strong connection between the largest movements in the exchange rate returns and the macroeconomic fundamentals. Given the difficulty in finding a relationship between the fundamentals and the exchange rates, it might very well be that the more extreme movements in the exchange rate returns are also unrelated to the conventional macroeconomic fundamental. Within the realm of a model like the monetary model of the exchange rate this means that the larger forex movements are purely due exogenous noise. To determine whether the extreme linkage exists, in this article we examine the asymptotic dependency between the variables.

While exchange rate returns are well known to have distributions with heavy tails, we first ask whether this can also be the case for the fundamentals that are supposedly driving the exchange rate returns. Specifically, we consider multiplicative parameter uncertainty in a standard monetary macroeconomic model. We show that the unconditional distributions of macroeconomic variables like money supply, interest rate and inflation rate can be heavy tailed, even if the innovations are not. Since standard models of the exchange rate imply that the exchange rate returns are driven by the growth rates of the macroeconomic fundamentals, using statistical extreme value theory we argue that the exchange rate returns are heavy-tailed distributed by implication. Subsequently, we demonstrate that the heavy tails on both sides of the linear exchange rate models imply asymptotic dependence between the exchange rate returns and the macroeconomic shocks.

To see whether there in fact exists such a connection, or that the larger movements in the exchange rate returns are exclusively due to exogenous noise, in our empirical investigation we use a data set consisting of monthly observations from 34 countries from February 1974 to May 2016. To deal with the small number of observations resulting from the low frequency of macroeconomic variables, we pool the data by region: combining European, Asian and Latin American countries into three groups. Based on the estimates of the tail shape parameter, the Asian and Latin American exchange rate returns and some of the macroeconomic fundamentals clearly have heavy tails. Moreover, the currency depreciations of the Asian and Latin American countries and the lagged monetary variables are significantly asymptotically dependent. With a probability of roughly more than 20%, the large downward swings in currency prices are likely to be proceeded by large movements in the monetary fundamentals, like money supply, price and interest rate.

No such dependency was detected for European currencies, whether we use the US dollar, the German mark or the euro as an anchor. In addition, real income does not show tail dependency with nominal exchange rates for any of the three regions. This is also in line with the theoretical analysis, which shows that the monetary fundamentals may be tail dependent with exchange rate returns, but this does not apply to real income. Therefore, we may conclude that the heavy tail feature of the FX returns is, at least partially, attributable to the tail behavior of the macroeconomic fundamentals. The responses of the exchange rate returns to large changes in macroeconomic fundamentals, however, are asymmetric. The asymptotic dependence is only significant for the depreciation of the domestic currency. The paper thereby lends support to traditional exchange rate models connecting the largest depreciations to movements in the fundamentals.

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Appendix A: Data and Descriptive Statistics

The data are monthly observations on the exchange rate, money supply (M2), production index, interest rate and price index from the IMF International Financial Statistics (IFS). The variables are relative to the US and the data ranges from February 1974 to May 2016. For the European countries, we also consider variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). For the countries that adopt the euro, the data ends in December 1998.

The list of 34 countries used in our study consists of Argentina, Austria, Bolivia, Brazil, Chile, China, Colombia, Denmark, Ecuador, Finland, France, Germany, India, Indonesia, Ireland, Israel, Italy, Japan, Jordan, Malaysia, Mexico, Netherlands, Norway, Pakistan, Peru, Philippines, Singapore, South Korea, Spain, Sweden, Turkey, UK, Uruguay and Venezuela.

Table A1 provides descriptive statistics for the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi, for Europe, Asia and Latin America, respectively. The Jarque-Bera (J-B) normality test rejects the null hypothesis of a normal population distribution at the 1% significance level (*p-values* equal zero) in all cases, while the skewness and kurtosis values indicate non-normal distributions of the variables. The kurtosis is particularly high for the exchange rate returns and most of the fundamentals from Asia and Latin America. A high kurtosis can be indicative of heavy tails.

Appendix B: Regular Variation and Tail Additivity

The monetary-approach exchange rate model is linear in the macroeconomic fundamentals. Suppose that the distributions of the macroeconomic variables exhibit heavy tails. We first show that if the macroeconomic variables are i.i.d., then the exchange rate also has a distribution with heavy tails. Subsequently, we argue that this result still follows if the macroeconomic variables are (cross sectionally) dependent. From an economic point of view the independence case is, in a way, the hardest case to treat. Since if, say, the macroeconomic fundamentals are driven by a common component that is heavy-tailed distributed, then it is almost immediate that this property is transferred to the distribution of the exchange rate.

We adopt the following general notion of heavy tails. A distribution function $F\left(q\right)$ is said to exhibit heavy tails if its tails vary regularly at infinity. The upper tail varies regularly at infinity with tail index α if [18]

$$\lim_{t\to\infty}\frac{1-F(tq)}{1-F(t)}=q^{-\alpha},\ q>0\text{ and }\alpha>0. \tag{31}$$

Regular variation implies that the tail of the distribution changes at a power rate. This contrasts with, e.g., the normal distribution that has tail probabilities that decline at an exponential rate. The number of bounded moments of $F(.)$ is finite and equals the integer value of α, i.e. the α-moment.[19] One checks that the Student-t distribution satisfies equation (31) by using L'Hôpital's rule and the expression for the density, for the Pareto distribution this is trivial.

Random variables with regularly varying distributions satisfy an important additivity property in the tail area. Suppose a distribution has heavy tails, so that

$$\Pr\{X>q\}=1-F(q)\sim Aq^{-\alpha},\ as\ q\to\infty. \tag{32}$$

According to Feller's Convolution Theorem (1971, VIII.8), if X_1 and X_2 are i.i.d. with c.d.f. $F(q)$ which has regularly varying tails as in equation (32), then

$$\Pr\{X_1+X_2>q\}\sim 2Aq^{-\alpha},\ as\ q\to\infty. \tag{33}$$

This result says that in the tail area the probability of the sum of random variables is equal to the sum of the marginal probabilities. If X and Y are i.i.d. and if X has a tail index of α and Y has a lighter tail (e.g. has a hyperbolic tail with a higher power than α or even has an exponential type tail), then analogous to the proof of (12) one shows that

$$\Pr\{X+Y>q\}\sim Aq^{-\alpha}. \tag{34}$$

In this case the convolution is dominated by marginal distribution of the heavier tail.

Some intuition for the Feller theorem is as follows. Let X be i.i.d. Pareto distributed with scale $A=1$. Then for large q

$$1-\Pr\{X_1\le q,X_2\le q\}=1-\left(1-q^{-\alpha}\right)^2\approx 2q^{-\alpha}$$

[18] For the lower tail, $\lim_{t\to\infty}F(-tq)/F(-t)=q^{-\alpha}$, $q>0$ and $\alpha>0$.

[19] For instance, the Pareto distribution satisfies the Power law and has a number of bounded moments equal to an integer of α. The Student-t distribution has moments equal to its degree of freedom. Per contrast, the thin-tailed normal distribution has all moments bounded.

since the term $q^{-2\alpha}$ is of smaller order. This is why only the (univariate) probability mass along the axes counts. To a first order, the probability mass above the line $X_1 + X_2 = q$ is also determined by how much probability mass is aligned along the axes above this line, i.e. $2q^{-\alpha}$. The probability mass above the line away from the axes is of smaller order. To see this, note that for any $\lambda\epsilon(0,1)$

$$\Pr\{\lambda X_1 > q, (1-\lambda) X_2 > q\} = \Pr\{\lambda X_1 > q\} \Pr\{(1-\lambda) X_2 > q\} = O\left(q^{-2\alpha}\right).$$

The convolution result (33) and (34) are very powerful. To give an illustrative example, consider the quasi-reduced-form specification of the exchange rate models in the logarithmic form

$$\Delta s_t = \varphi_1 \Delta m_{t-1} + \varphi_2 \Delta y_{t-1} + \xi_t, \tag{35}$$

that is behind equation (19) from the main text. The ξ_t is the composite of the shocks.

The convolution theorem holds that if the distributions of Δm_{t-1}, Δy_{t-1} and ξ_t adhere to equation (31), are independent and

$$\alpha_s = \alpha_m = \alpha_y = \alpha_\xi, \tag{36}$$

then

$$\begin{aligned} \Pr\{\Delta s_t > t\} &= P\{\varphi_1 \Delta m_{t-1} + \varphi_2 \Delta y_{t-1} + \xi_t > t\} \\ &\sim (\varphi_1^\alpha + \varphi_2^\alpha + 1)\, t^{-\alpha}. \end{aligned} \tag{37}$$

If, however, for example

$$\alpha_s = \alpha_m = \alpha_\xi < \alpha_y$$

then

$$\alpha_s = \min(\alpha_m, \alpha_y, \alpha_\xi) = \alpha_m = \alpha_\xi.$$

We find that the tail shape of the exchange rate returns Δs is governed by the tail shape of the fundamentals or the noise distributions with the heaviest tail.

The convolution results (33) and (37) assume that the macroeconomic variables from equation (35) are independent random variables. This is often not the case due to endogeneity. Consider therefore the multivariate extension of (31). Suppose that the vector $\mathbf{x}$ of fundamental variables is multivariate regularly varying in the sense that

$$\lim_{t\to\infty} \frac{1 - F(t\mathbf{x})}{1 - F(t\mathbf{1})} = W(\mathbf{x}),\ \mathbf{x} > 0,$$

where $W(.)$ is a function such that $W(\lambda\mathbf{x}) = \lambda^{-\alpha} W(\mathbf{x})$, $\alpha > 0$, $\lambda > 0$ and $\mathbf{1}$ is the unit vector. Suppose the marginal distributions are as in equation (32) so that the scales are of the same order, and all the marginal distributions have the same tail index α. Then for any non-zero weight vector $\mathbf{w}$, $P\{\mathbf{w}^T\mathbf{x} > q\} \sim Cq^{-\alpha}$, as $q \to \infty$. Here the scale constant C depends on the type of dependence and can no longer be determined as in equation (37), i.e. it requires specific knowledge of the copula. Nevertheless, the weighted sum of macroeconomic variables that determines the distribution of the exchange rate still has a Pareto like upper tail with the tail index α. Moreover, it is still the case that if the marginal distributions have different tail indices, the fundamental with the heaviest tail determines the tail index of the exchange rate returns.

In addition, we like to note that the a-temporal convolution result still holds when the economic variables are stationary time series. This is so since the convolution is a ‘cross-section’ like aggregation at a specific point in time. As the exchange rate and macroeconomic variables display bouts of quiescence and turbulence, changes in the economic variables are often captured by ARMA-GARCH type of models. From the convolution result (33), one can show that when time series are not i.i.d. but serially dependent, the occurrence of extremes may affect the distribution of order statistics, but not the tail index α. That is the exchange rate return distribution still has hyperbolic tails.

The convolution theorem can nevertheless also be used to study the aggregation of time series over time. Suppose for example that m follows the following $MA(1)$ process

$$m_t = \varepsilon_t + \delta\varepsilon_{t-1}, \text{ and } \delta > 0,$$

and where the innovations ε are i.i.d. with distribution function as in equation (32). Then, by Feller's Convolution Theorem

$$\Pr\{m > q\} \sim A\,(1 + \delta^{\alpha})\,q^{-\alpha},\ as\ q \to \infty.$$

Furthermore, $P\{m_t + m_{t-1} > q\} \sim A\,[1 + (1+\delta)^{\alpha} + \delta^{\alpha}]\,q^{-\alpha}$, $as\ q \to \infty$. Note that the convolution results show that the scales of the random variables change due to the moving average process, but not the tail index α.

More complicated time series models can also be handled. For instance, Engle's (1982) original contribution modeled the inflation rate by the ARCH process. De Haan et al. (1989) showed that the tail of the stationary distribution of the ARCH process is regularly varying. Basrak, Davis and Mikosch (2002) discuss the convolution of GARCH processes.

Table A1: Descriptive Statistics

Europe (DM/EUR)	Δs	Δm	Δy	Δp	Δi
Mean	0.0018	0.0013	0.0007	0.0019	1.5E-06
Median	0.0003	0.0003	-0.0005	0.0014	0.0000
Max	0.2208	0.5703	1.0209	0.0423	0.0581
Min	-0.1036	-0.2490	-1.1087	-0.0282	-0.0545
Std. Dev	0.0168	0.0252	0.1504	0.0055	0.0019
Skewness	1.9916	3.5906	0.0006	1.0502	1.2168
Kurtosis	21.5115	82.5704	17.1546	8.1680	458.8525
J-B test	59099.88	1022081	32932.90	5129.58	33500286
Obs.	3956	3843	3945	3956	3869
Europe (USD)	Δs	Δm	Δy	Δp	Δi
Mean	0.0006	0.0017	-0.0003	0.0007	3.25E-07
Median	-0.0007	0.0002	0.0035	9.70E-05	0.0000
Max	0.1991	0.3202	0.9380	0.0688	0.0578
Min	-0.1314	-0.2468	-1.0013	-0.0535	-0.0547
Std. Dev	0.0314	0.0211	0.1582	0.0060	0.0017
Skewness	0.4043	1.2061	-0.3598	1.3394	1.1433
Kurtosis	4.9101	23.6552	16.6329	14.7690	575.1924
J-B test	793.04	74346.42	34106.13	2.22E+08	59015541
Obs.	4424	4126	4392	4414	4326
Asia	Δs	Δm	Δy	Δp	Δi
Mean	0.0055	0.0093	0.0023	0.0050	-2.36E-07
Median	0.0000	0.0055	0.0023	0.0015	-8.33E-06
Max	0.8025	1.4456	2.1472	0.2413	0.2934
Min	-0.3487	-0.6980	-2.0442	-0.0697	-0.3972
Std. Dev	0.0350	0.0355	0.1123	0.0171	0.0107
Skewness	5.9947	10.7632	0.1050	4.4266	-6.3496
Kurtosis	101.0635	522.6942	120.1407	38.1431	672.3097
J-B test	2479091	64209454	2997110	320309.5	1.08E+08
Obs.	6096	5696	5242	5853	5801
Latin America	Δs	Δm	Δy	Δp	Δi
Mean	0.0201	0.0282	-0.0002	0.0265	-2.33E-05
Median	0.0048	0.0153	-0.0024	0.0089	-4.07E-20
Max	28.4343	1.1846	1.8877	1.5942	69.4261
Min	-28.5646	-0.8963	-1.6044	-0.1062	-116.4454
Std. Dev	0.9182	0.0702	0.0872	0.0651	2.8090
Skewness	-6.0807	3.3705	1.0977	7.9626	-19.4327
Kurtosis	923.7547	52.6444	95.5615	118.3485	1228.61
J-B test	1.75E+08	514447.1	1378379	2791999	2.97E+08
Obs.	4967	4919	3859	4942	4745

Table A1 shows descriptive statistics for the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi, for Europe, Asia and Latin America. The variables are relative to the US and the US dollar (USD) is a base currency. However, for the group of European countries we also consider the domestic variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The Deutsche mark (DM), and thereafter the euro (EUR) are then a base currency.

Appendix C: Results of Robustness Checks

Table A2: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals

European Currencies (DM/EUR)

Lag i	1	2	3	4	5	6	7	8	9	10	11	12
Δs, Δm_{-i}, +	0.0140	0.0521	0.0521	0.0104	0.0000	0.0313	0.0421	0.0000	0.0632	0.0526	0.0211	0.0421
	(-0.0250, 0.0458)	(-0.0254, 0.1296)	(-0.0238, 0.1280)	(-0.0251, 0.0460)	(-0.0049, 0.0049)	(-0.0310, 0.0935)	(-0.0281, 0.1123)	(-0.0097, 0.0097)	(-0.0199, 0.1462)	(-0.0246, 0.1299)	(-0.0288, 0.0709)	(-0.0258, 0.1101)
Δs, Δy_{-i}, -	0.0510	0.0204	0.0510	0.0408	0.0102	0.0510	0.0714	0.0204	0.0306	0.0000	0.0408	0.0102
	(-0.0217, 0.1237)	(-0.0273, 0.0681)	(-0.0246, 0.1267)	(-0.0264, 0.1080)	(-0.0243, 0.0447)	(-0.0238, 0.1259)	(-0.0159, 0.1587)	(-0.0283, 0.0691)	(-0.0290, 0.0902)	(-0.0074, 0.0074)	(-0.0280, 0.1097,)	(-0.0242, 0.0447)
Δs, Δp_{-i}, +	0.0404	0.0606	0.0612	0.0510	0.0612	0.0408	0.0306	0.0408	0.0510	0.0510	0.0510	0.0510
	(-0.0269, 0.1077)	(-0.0174, 0.1386)	(-0.0216, 0.1440)	(-0.0238, 0.1259)	(-0.0181, 0.1406)	(-0.0254, 0.1070)	(-0.0273, 0.0886)	(-0.0269, 0.1086)	(-0.0238, 0.1259)	(-0.0226, 0.1246)	(-0.0226, 0.1246)	(-0.0246, 0.1267)
Δs, Δi_{-i}, +	0.0928	0.0729	0.0417	0.0729	0.1146	0.0729	0.0521	0.0208	0.0417	0.0417	0.0625	0.0625
	(-0.0126, 0.1982)	(-0.0185, 0.1644)	(-0.0287, 0.1120)	(-0.0185, 0.1644)	(0.0003, 0.2289)	(-0.0166, 0.1624)	(-0.0249, 0.1291)	(-0.0288, 0.0705)	(-0.0301, 0.1135)	(-0.0287, 0.1120)	(-0.0236, 0.1486)	(-0.0223, 0.1473)
Δs, Δi_{-i}, -	0.0309	0.0625	0.0729	0.0833	0.0313	0.0625	0.0625	0.0625	0.0625	0.0521	0.0521	0.0417
	(-0.0280, 0.0898)	(-0.0236, 0.1486)	(-0.0166, 0.1624)	(-0.0143, 0.1810)	(-0.0310, 0.0935)	(-0.0205, 0.1455)	(-0.0185, 0.1435)	(-0.0197, 0.1447)	(-0.0236, 0.1486)	(-0.0238, 0.1280)	(-0.0249, 0.1291)	(-0.0273, 0.1106)

European Currencies (USD)

Lag i	1	2	3	4	5	6	7	8	9	10	11	12
Δs, Δm_{-i}, +	0.0194	0.0291	0.0971	0.0097	0.0194	0.0194	0.0388	0.0097	0.0291	0.0388	0.0097	0.0000
	(-0.0263, 0.0652)	(-0.0272, 0.0855)	(-0.0039, 0.1980)	(-0.0231, 0.0426)	(-0.0268, 0.0656)	(-0.0278, 0.0666)	(-0.0253, 0.1030)	(-0.0229, 0.0423)	(-0.0266, 0.0848)	(-0.0262, 0.1039)	(-0.0231, 0.0426)	(-0.0082, 0.0082)
Δs, Δy_{-i}, -	0.0364	0.0364	0.0550	0.0367	0.0183	0.0183	0.0642	0.0275	0.0275	0.0092	0.0275	0.0367
	(-0.0254, 0.0982)	(-0.0245, 0.0972)	(-0.0207, 0.1307)	(-0.0234, 0.0968)	(-0.0255, 0.0622)	(-0.0255, 0.0622)	(-0.0163, 0.1448)	(-0.0261, 0.0811)	(-0.0261, 0.0811)	(-0.0244, 0.0428)	(-0.0257, 0.0808)	(-0.0263, 0.0997)
Δs, Δp_{-i}, +	0.0636	0.0455	0.0091	0.0091	0.0091	0.0636	0.0000	0.0091	0.0364	0.0455	0.0091	0.0273
	(-0.0181, 0.1454)	(-0.0213, 0.1123)	(-0.0225, 0.0407)	(-0.0215, 0.0397)	(-0.0217, 0.0398)	(-0.0156, 0.1429)	(-0.0053, 0.0053)	(-0.0217, 0.0398)	(-0.0239, 0.0967)	(-0.0218, 0.1127)	(-0.0225, 0.0407)	(-0.0245, 0.0790)
Δs, Δi_{-i}, +	0.0833	0.0185	0.0093	0.0093	0.0278	0.0185	0.0370	0.0833	0.0278	0.0278	0.0463	0.0463
	(-0.0095, 0.1761)	(-0.0251, 0.0621)	(-0.0228, 0.0413)	(-0.0224, 0.0409)	(-0.0257, 0.0813)	(-0.0259, 0.0630)	(-0.0259, 0.1000)	(-0.0129, 0.1796)	(-0.0279, 0.0834)	(-0.0279, 0.0834)	(-0.0208, 0.1134)	(-0.0249, 0.1175)
Δs, Δi_{-i}, -	0.0648	0.0833	0.0556	0.0556	0.0185	0.0093	0.0093	0.0278	0.0556	0.0648	0.0278	0.0278
	(-0.0184, 0.1481)	(-0.0110, 0.1777)	(-0.0193, 0.1304)	(-0.0200, 0.1311)	(-0.0251, 0.0621)	(-0.0218, 0.0403)	(-0.0241, 0.0426)	(-0.0263, 0.0818)	(-0.0231, 0.1342)	(-0.0162, 0.1458)	(-0.0279, 0.0834)	(-0.0257, 0.0813)

Table A2: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals (Continued)

Asian Currencies (USD)

Lag i	1	2	3	4	5	6	7	8	9	10	11	12
Δs, Δm_{-i}, +	0.2676	0.1888	0.2113	0.1972	0.1761	0.1972	0.1901	0.1479	0.1761	0.1338	0.1761	0.1620
	(0.1437, 0.3915)	(0.0856, 0.2920)	(0.1002, 0.3223)	(0.0882, 0.3061)	(0.0752, 0.2769)	(0.0806, 0.3137)	(0.0739, 0.3064)	(0.0508, 0.2450)	(0.0701, 0.2820)	(0.0418, 0.2259)	(0.0703, 0.2818)	(0.0625, 0.2614)
Δs, Δy_{-i}, -	0.0229	0.0153	0.0153	0.0229	0.0153	0.0229	0.0153	0.0000	0.0153	0.0154	0.0231	0.0231
	(-0.0215, 0.0673)	(-0.0208, 0.0513)	(-0.0208, 0.0513)	(-0.0218, 0.0676)	(-0.0213, 0.0518)	(-0.0218, 0.0676)	(-0.0213, 0.0518)	(-0.0077, 0.0077)	(-0.0213, 0.0518)	(-0.0210, 0.0517)	(-0.0211, 0.0673)	(-0.0211, 0.0673)
Δs, Δp_{-i}, +	0.2466	0.2329	0.2260	0.2260	0.2534	0.2192	0.1986	0.1986	0.2329	0.2260	0.1986	0.1986
	(0.1337, 0.3594)	(0.1204, 0.3453)	(0.1173, 0.3347)	(0.1194, 0.3327)	(0.1404, 0.3665)	(0.1130, 0.3254)	(0.0924, 0.3048)	(0.0948, 0.3024)	(0.1211, 0.3447)	(0.1102, 0.3418)	(0.0874, 0.3099)	(0.0905, 0.3067)
Δs, Δi_{-i}, +	0.1862	0.1793	0.1517	0.1793	0.1448	0.1586	0.1724	0.1517	0.1448	0.1724	0.1448	0.1034
	(0.0675, 0.3049)	(0.0664, 0.2923)	(0.0492, 0.2543)	(0.0725, 0.2861)	(0.0415, 0.2482)	(0.0507, 0.2665)	(0.0632, 0.2816)	(0.0446, 0.2589)	(0.0431, 0.2465)	(0.0582, 0.2866)	(0.0470, 0.2426)	(0.0185, 0.1884)
Δs, Δi_{-i}, -	0.1034	0.0966	0.0621	0.0690	0.1103	0.0828	0.0759	0.0759	0.0621	0.0690	0.0621	0.0414
	(0.0174, 0.1895)	(0.0134, 0.1797)	(-0.0033, 0.1274)	(-0.0001, 0.1380)	(0.0175, 0.2032)	(0.0077, 0.1578)	(0.0036, 0.1482)	(-0.0011, 0.1529)	(-0.0043, 0.1284)	(-0.0017, 0.1396)	(-0.0076, 0.1317)	(-0.0133, 0.0960)

Latin American Currencies (USD)

Lag i	1	2	3	4	5	6	7	8	9	10	11	12
Δs, Δm_{-i}, +	0.2276	0.2195	0.1870	0.1707	0.1626	0.1382	0.1545	0.1707	0.1707	0.1707	0.2033	0.1707
	(0.1121, 0.3432)	(0.1056, 0.3334)	(0.0791, 0.2948)	(0.0649, 0.2765)	(0.0599, 0.2654)	(0.0405, 0.2359)	(0.0530, 0.2559)	(0.0667, 0.2747)	(0.0667, 0.2748)	(0.0658, 0.2756)	(0.0938, 0.3127)	(0.0655, 0.2759)
Δs, Δy_{-i}, -	0.0417	0.0208	0.0208	0.0104	0.0000	0.0103	0.0000	0.0000	0.0206	0.0309	0.0000	0.0103
	(-0.0287, 0.1120)	(-0.0300, 0.0717)	(-0.0281, 0.0698)	(-0.0256, 0.0464)	(-0.0031, 0.0031)	(-0.0248, 0.0455)	(-0.0059, 0.0059)	(-0.0059, 0.0059)	(-0.0283, 0.0696)	(-0.0299, 0.0918)	(-0.0059, 0.0059)	(-0.0252, 0.0459)
Δs, Δp_{-i}, +	0.2358	0.2764	0.2358	0.2276	0.2033	0.2195	0.2114	0.2683	0.2276	0.2114	0.2033	0.2276
	(0.1213, 0.3502)	(0.1539, 0.3989)	(0.1219, 0.3496)	(0.1121, 0.3432)	(0.0906, 0.3159)	(0.1049, 0.3341)	(0.0974, 0.3253)	(0.1455, 0.3911)	(0.1135, 0.3418)	(0.0977, 0.3250)	(0.0935, 0.3130)	(0.1114, 0.3439)
Δs, Δi_{-i}, +	0.2373	0.2203	0.2203	0.1864	0.1610	0.1780	0.1864	0.1780	0.2203	0.2034	0.1695	0.2034
	(0.1156, 0.3590)	(0.1015, 0.3392)	(0.1055, 0.3352)	(0.0761, 0.2968)	(0.0530, 0.2691)	(0.0691, 0.2868)	(0.0708, 0.3021)	(0.0682, 0.2877)	(0.1008, 0.3399)	(0.0888, 0.3180)	(0.0600, 0.2790)	(0.0907, 0.3161)
Δs, Δi_{-i}, -	0.1017	0.1356	0.1102	0.1186	0.1102	0.1102	0.1525	0.1441	0.1186	0.1102	0.1186	0.0932
	(0.0049, 0.1985)	(0.0254, 0.2458)	(0.0169, 0.2035)	(0.0211, 0.2162)	(0.0151, 0.2053)	(0.0164, 0.2040)	(0.0379, 0.2672)	(0.0363, 0.2518)	(0.0209, 0.2164)	(0.0159, 0.2044)	(0.0211, 0.2162)	(0.0024, 0.1840)

Table A2 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables (from lag 1 to lag 12), and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, Δs, Δm_{-i}, + represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to abroad. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

Table A3: Estimates of Extreme Linkage between Macroeconomic Fundamentals and Lagged Currency Depreciations

	Europe	Europe	Asia	Latin America
Base Currency	**DM/EUR**	**USD**	**USD**	**USD**
Δm, Δs_{-1}, +	0.0313	0.0388	0.2394	0.2520
	(-0.0297, 0.0922)	(-0.0244, 0.1021)	(0.1273, 0.3516)	(0.1351, 0.3690)
Δy, Δs_{-1}, -	0.0408	0.0273	0.0305	0.0313
	(-0.0280, 0.1097)	(-0.0252, 0.0798)	(-0.0210, 0.0821)	(-0.0285, 0.0910)
Δp, Δs_{-1}, +	0.0707	0.0818	0.3151	0.2846
	(-0.0134, 0.1548)	(-0.0062, 0.1699)	(0.1822, 0.4480)	(0.1629, 0.4062)
Δi, Δs_{-1}, +	0.0619	0.0556	0.1724	0.1864
	(-0.0242, 0.1479)	(-0.0231, 0.1342)	(0.0599, 0.2849)	(0.0783, 0.2946)
Δi, Δs_{-1}, -	0.0825	0.0556	0.1310	0.1102
	(-0.0114, 0.1764)	(-0.0189, 0.1300)	(0.0313, 0.2307)	(0.0140, 0.2063)

Table A3 shows the estimates of the extreme linkage between *time t* macroeconomic variables and *time t-1* domestic currency depreciations, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. For instance, Δm, Δs_{-1}, + represents the linkage between the right tails of the distributions of *time t* relative money supply growth and *time t-1* exchange rate returns. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

Table A4: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals, using quarterly data

	Europe	Europe	Asia	Latin America
Base Currency	**DM/EUR**	**USD**	**USD**	**USD**
Δs, Δm_{-1}, +	0.0625	0.0294	0.3191	0.2927
	(-0.0861, 0.2112)	(-0.0820, 0.1408)	(0.1065, 0.5318)	(0.0853, 0.5001)
Δs, Δy_{-1}, -	0.0313	0.0000	0.0233	0.0938
	(-0.0714, 0.1339)	(-0.0095, 0.0095)	(-0.0552, 0.1017)	(-0.0825, 0.2700)
Δs, Δp_{-1}, +	0.0909	0.0556	0.3750	0.3659
	(-0.0836, 0.2654)	(-0.0818, 0.1929)	(0.1622, 0.5878)	(0.1492, 0.5825)
Δs, Δi_{-1}, +	0.0313	0.1944	0.2292	0.3590
	(-0.0723, 0.1348)	(-0.0238, 0.4127)	(0.0252, 0.4331)	(0.1213, 0.5967)
Δs, Δi_{-1}, -	0.0938	0.0556	0.1250	0.1282
	(-0.0825, 0.2700)	(-0.0746, 0.1857)	(-0.0399, 0.2899)	(-0.0651, 0.3215)

Table A4 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The variables are *quarterly*. They are the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, Δs, Δm_{-1}, + represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to abroad. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

Table A5: Estimates of Extreme Linkage between Currency Depreciations and Lagged Macroeconomic Fundamentals, excluding crisis episodes

	Whole sample	**Whole sample**
	(1) Incl. crises obs.	**(2) Excl. crises obs.**
Δs, Δm_{-1}, +	0.3913	0.0647
	(0.3009, 0.4817)	(0.0158, 0.1137)
Obs.	12881	11124
Δs, Δy_{-1}, -	0.0205	0.0307
	(-0.0078, 0.0487)	(-0.0055, 0.0668)
Obs.	11705	10448
Δs, Δp_{-1}, +	0.4377	0.0737
	(0.3483, 0.5271)	(0.0224, 0.1250)
Obs.	13160	11390
Δs, Δi_{-1}, +	0.3220	0.0599
	(0.2325, 0.4114)	(0.0123, 0.1074)
Obs.	13372	11362
Δs, Δi_{-1}, -	0.1362	0.0528
	(0.0730, 0.1994)	(0.0089, 0.0967)
Obs.	12929	11362

Table A5 shows the estimates of the extreme linkage between domestic currency depreciations and lagged macroeconomic variables, and asymptotic 95% confidence intervals (in parentheses) using 2.5% of the data. The purpose is to demonstrate that the results of extreme linkages are influenced by crisis episodes. The left column (1) shows the estimates using the entire observations, while the right column (2) shows the results when excluding crisis episodes, dubbed freely falling regime, classified in Ilzetzki, Reinhart and Rogoff (2008). The variables are the exchange rate returns Δs, relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi. The first column shows the pair of variables under investigation. Positive and negative signs indicate positive and negative relations between the two variables. The depreciation side is on the right tail of the exchange rate returns distribution. Hence, Δs, Δm_{-1}, + represents the linkage between the right tails of the distributions of exchange rate returns and lagged money supply growth, i.e. between large depreciations of the domestic currency and increases in domestic money supply relative to abroad. For all three groups, i.e. Europe, Asia and Latin America, the US dollar (USD) is a base currency and the domestic fundamentals are relative to the US. However, for the European countries we also consider the variables relative to Germany and the Euro zone (after the introduction of the euro in 1999). The deutsche mark (DM), and subsequently the euro (EUR) are a base currency.

Figure A1: Plots of the conditional probability

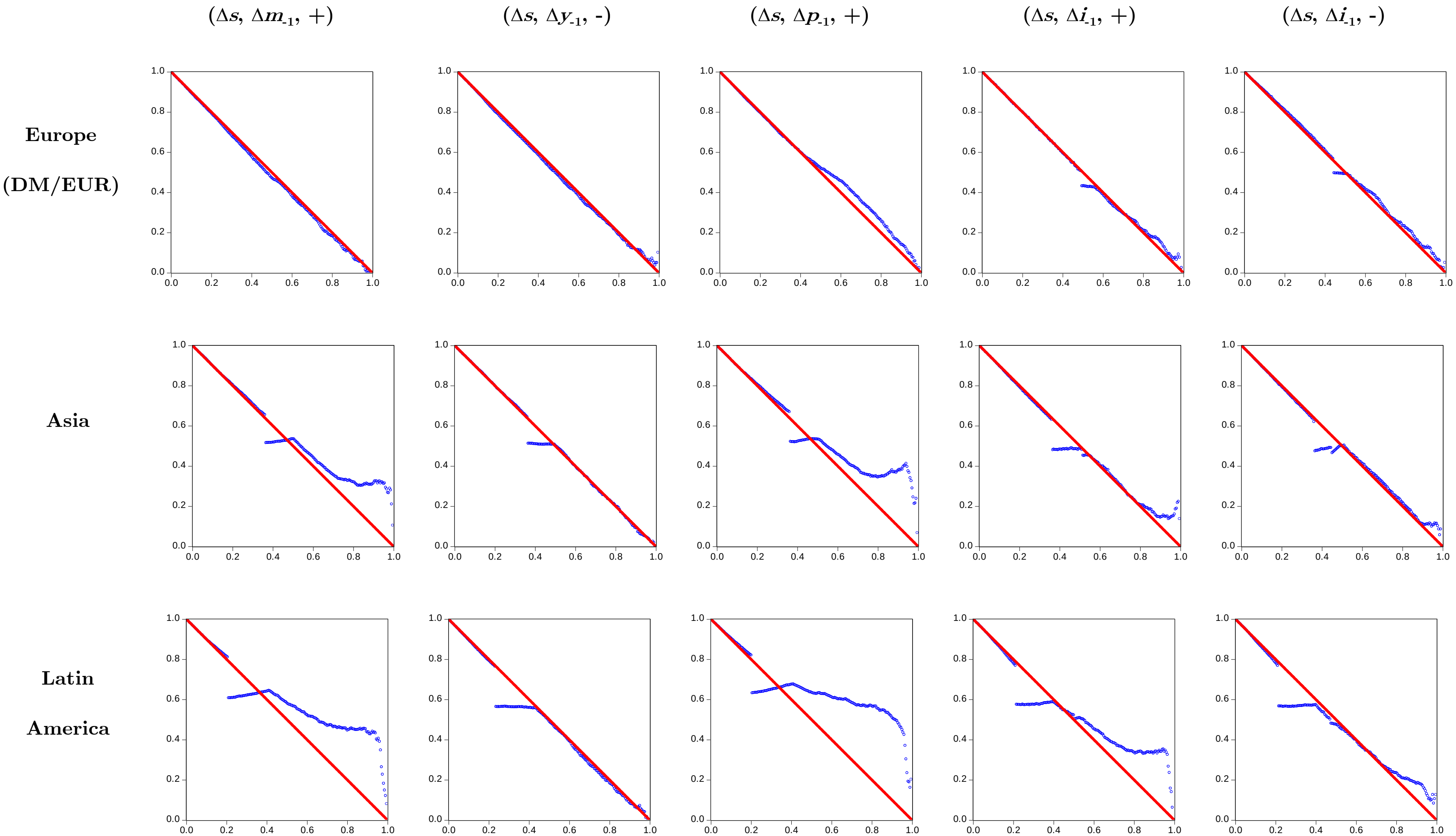

Figure A1 shows the plots of the conditional probability Pr($\Delta s > q_s(p) \mid \Delta x_{-1} > q_x(p)$) of large currency depreciations Δs given large shocks on macroeconomic variable Δx_{-1} (on the Y-axis) as a function of *1*-p, with *p* the percentile of the variable Δx_{-1}. That is when the shock becomes large $q_x(p) \rightarrow \infty$, on the X-axis *1-p*→1. The macroeconomic variables are relative money supply growth Δm, relative real income growth Δy, relative inflation Δp and changes in the interest rate differential Δi.